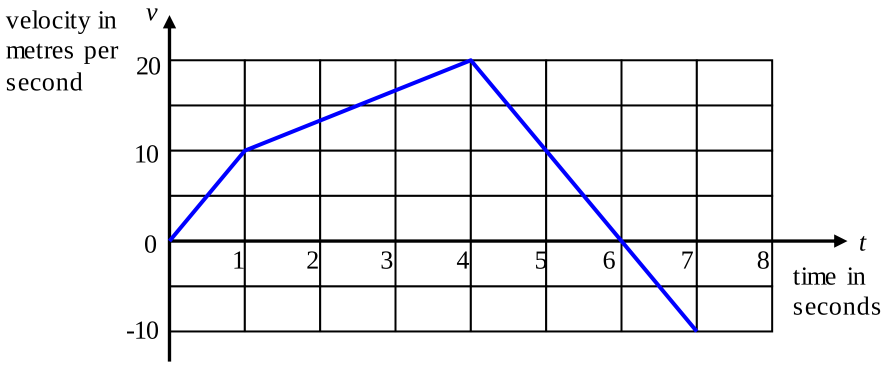
Topic assessment

1. A particle travels in a straight line.
   The motion is modelled by the v-t diagram below.



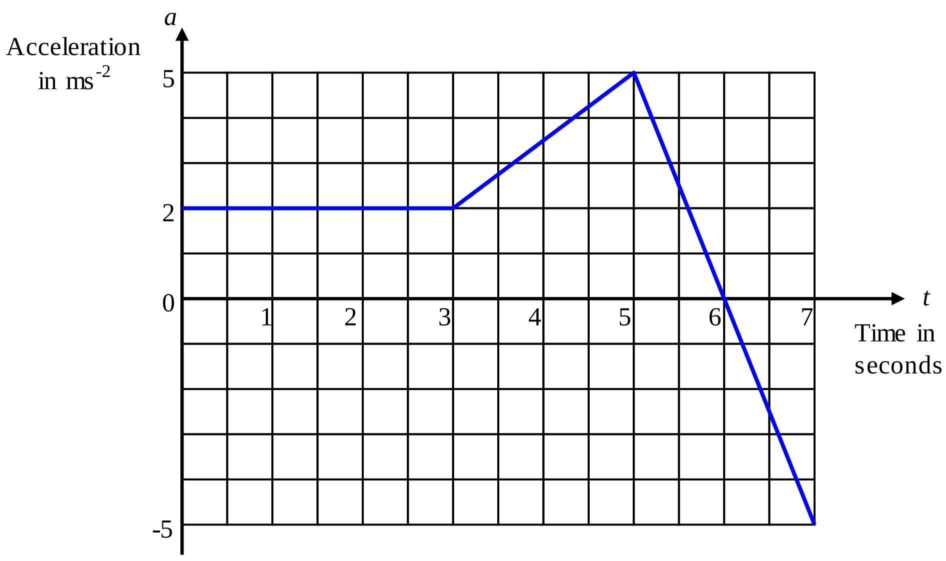
(i) Calculate the acceleration of the particle in the part of the motion from $t = 1$ to $t = 4$. [2]

(ii) Calculate the displacement of the particle from its position when $t = 0$ to its position when $t = 6$. [4]

(iii) Calculate the displacement of the particle from its position when $t = 0$ to its position when $t = 7$. [2]

(iv) Describe the motion of the particle during the interval $4 \le t \le 7$. [2]

2.



A car is travelling due east along a straight road when it passes a point P. The *acceleration* of the car during the next 7 seconds is modelled in the acceleration-time graph above, where a ms^{-2} is the acceleration of the car due east and t seconds is the time after passing the point P.

(i) Explain why the speed of the car is greatest when $t = 6$. [1]

The speed of the car when it passes P is 12 ms^{-1}.

(ii) Calculate the speed of the car when $t = 3$. [3]
(iii) Show that, when $t = 5$, the speed of the car is 25 ms^{-1}. [3]
(iv) Show that, for $5 \le t \le 7$, the acceleration is given by
$a = -5t + 30$. [2]

3. A ball A is thrown vertically upwards at 25 ms^{-1} from a point P. Three seconds later a second ball B is also thrown vertically upwards from the point P at 25 ms^{-1}. Taking the acceleration due to gravity to be 10 ms^{-2}, calculate

(i) the time for which ball A has been in motion when the balls meet [7]
(ii) The height above P at which A and B meet. [2]

4. Cars A and B are travelling in the same direction along a straight road. The time t is in seconds.

At $t = 0$, car A is at rest. It accelerates at 3 ms^{-2} for $0 \le t \le 10$ and then travels at a constant speed.

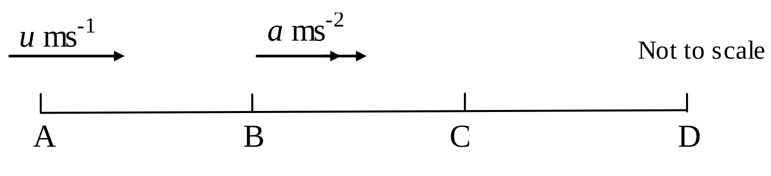
Car B travels at 15 ms^{-1} for $0 \le t \le 30$ and then accelerates at 1 ms^{-2} until it reaches a speed of 25 ms^{-1}, after which it continues at this constant speed.

(i) Draw v-t diagrams for the motion of car A and of car B, where v is the speed in ms^{-1} and $0 \le t \le 80$. [4]
(ii) Show that, in the first 40 seconds, car A travels 400 m further than car B. [4]
(iii) Given that car A is 500 m behind car B at $t = 0$, at what value of t does car A catch up with car B? [2]

5. (a) A particle accelerates uniformly from 7 ms^{-1} to 21 ms^{-1} in 8 s.
How far does it travel in this time? [2]

(b) Lewis is travelling in a car along a straight road. He wonders whether the car is accelerating uniformly.
Lewis estimates that the car takes
5 s to travel a distance of 75 m from A to B,
15 s to travel a distance of 315 m from A to C.

Lewis models the acceleration as a constant a ms^{-2}. He also takes the speed of the car at A to be u ms^{-1}, as shown in the diagram below.



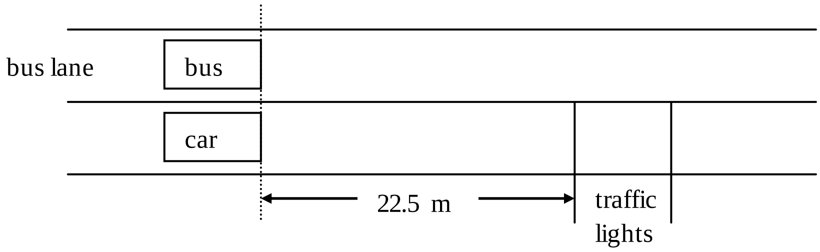
(i) By considering the motion from A to B, show that $75 = 5u + 12.5a$. [3]
(ii) Find a second equation involving u and a. [4]
(iii) Hence find the value of u and show that $a = 1.2$. [3]

Lewis decides to check whether his assumption of constant acceleration is consistent with the motion of the car after reaching C. He notes that when the car reaches D, the distance CD is 200 m and the car's speed is 36.5 ms^{-1}.

(iv) Does the extra information suggest that the constant acceleration model is reasonable? [3]

6. (a) A particle travelling in a straight line at 15 ms^{-1} is brought to rest with constant deceleration in a distance of 22.5 m. Show that the deceleration takes 3 seconds. [2]

(b) A car and a bus are travelling along a straight road towards traffic lights (see diagram below).



The traffic lights change at time $t = 0$, where t is in seconds. At this instant the car has a speed of 15 ms^{-1}. The car then

- decelerates uniformly to rest in 22.5 m (as in part (a)),
- waits at the traffic lights for 7 seconds,
- accelerates uniformly up to 15 ms^{-1} in 5 seconds,
- travels at 15 ms^{-1} down the road.

(i) Sketch a v-t diagram for the motion of the car in the interval $0 \le t \le 20$.
Calculate the distance travelled by the car in the interval $0 \le t \le 15$. [6]

When $t = 0$ the car is level with a bus which is travelling at a constant speed of 20 ms^{-1} along a bus lane. The bus is not required to stop at the traffic lights and continues at this speed down the road.

(ii) Show that the bus has travelled 240 m further than the car at the time that the car again reaches 15 ms^{-1}. [2]

At the instant that the car reaches its constant speed of 15 ms^{-1}, the bus begins to decelerate uniformly at 0.2 ms^{-2}.

(iii) It takes T seconds for the car to catch up with the bus after the bus begins to decelerate. Show that the car must travel $240 + 20T - 0.1T^2$ in this time.
Hence show that T satisfies the equation $T^2 - 50T - 2400 = 0$.
Find the speed of the bus when the car catches up with it. [7]

Total 70 marks

Edexcel AS Maths Kinematics Assessment solution

Solutions to Topic Assessment

1. (i) $\text{Acceleration} = \dfrac{\text{change in velocity}}{\text{time taken}}$

$$= \frac{10}{3} \text{ ms}^{-2}$$

[2]

(ii) Displacement = area under graph

$$= \left(\tfrac{1}{2}\times 1\times 10\right) + \left(\tfrac{1}{2}\times 3\times (10+20)\right) + \left(\tfrac{1}{2}\times 2\times 20\right)$$
$$= 70 \text{ m}$$

[4]

(iii) Displacement between $t = 6$ and $t = 7$ is $\tfrac{1}{2}\times 1\times -10 = -5$

Displacement between $t = 0$ and $t = 7$ is $70 - 5 = 65$ m.

[2]

(iv) The particle decelerates at a constant rate of 10 ms^{-2}. When $t = 6$ the particle is instantaneously at rest and thereafter the particle is travelling in the opposite direction.

[2]

2. (i) The acceleration is positive until $t = 6$, so the speed is increasing from $t = 0$ to $t = 6$. At $t = 6$ the acceleration is zero, and then it is negative, so the speed then begins to decrease.

[1]

(ii) Increase in speed = area under graph between $t = 0$ and $t = 3$

Increase in speed $= 3 \times 2 = 6$

Speed when $t = 3 = 12 + 6 = 18$ ms^{-1}.

[3]

(iii) Increase in speed between $t = 3$ and $t = 5$ is area under graph

Increase in speed $= \tfrac{1}{2}\times 2\times (2+5) = 7$

Speed at $t = 5$ is $18 + 7 = 25$ ms^{-1}.

[3]

(iv) Gradient of graph for $5 \le t \le 7 = -5$

When $t = 6$, $a = 0$

$(a - 0) = -5(t - 6)$

$a = -5t + 30$

[2]

3. (i) When the balls meet:

For ball A: $u = 25$, $a = -10$, $t = T$, $s = h$

For ball B: $u = 25$, $a = -10$, $t = T - 3$, $s = h$

$$s = ut + \tfrac{1}{2}at^2$$
$$h = 25T - 5T^2$$

$$s = ut + \tfrac{1}{2}at^2$$
$$h = 25(T-3) - 5(T-3)^2$$

$$25T - 5T^2 = 25(T-3) - 5(T-3)^2$$
$$5T - T^2 = 5(T-3) - (T-3)^2$$
$$5T - T^2 = 5T - 15 - T^2 + 6T - 9$$
$$6T = 24$$
$$T = 4$$

A has been in motion for 4 seconds when the balls meet.

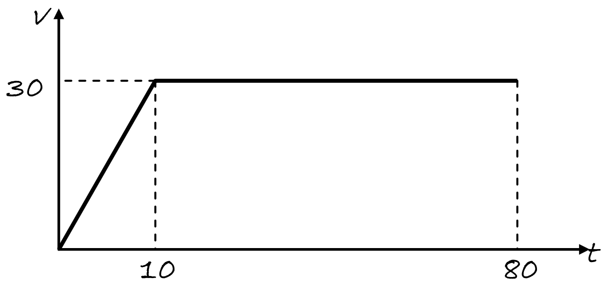
[7]

(ii) $h = 25T - 5T^2$
$= 25 \times 4 - 5 \times 4^2$
$= 100 - 80$
$= 20$

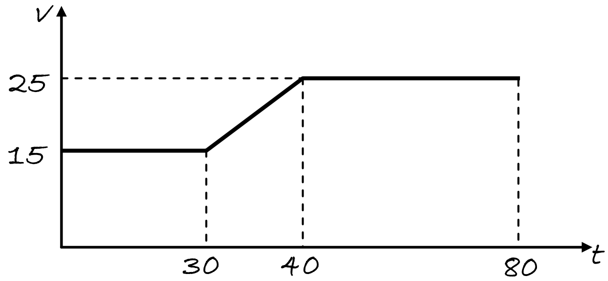
The balls meet 20 metres above P.

[2]

4. (i) Car A:



Car B:



[4]

(ii) In the first 10 seconds, car A travels $\frac{1}{2}\times 10\times 30 = 150$ m.
In the next 30 seconds, car A travels $30\times 30 = 900$ m.
Total distance travelled by A in the first 40 seconds = 1050 m.

In the first 30 seconds, car B travels $15\times 30 = 450$ m.
In the next 10 seconds, car B travels $\frac{1}{2}(15+25)\times 10 = 200$ m.
Total distance travelled by B in the first 40 seconds = 650 m.

So in the first 40 seconds, extra distance travelled by car A
= 1050 – 650 = 400 m.

[4]

(iii) After the first 40 seconds, car A is 100 m behind car B.
At this point car A is travelling at 30 ms^{-1} and car B is travelling at 25 ms^{-1}.
Distance travelled by A in t seconds is $30t$, and distance travelled by B is $25t$.
When A catches up with B, $30t - 25t = 100$

$$5t = 100$$
$$t = 20$$

so the extra time taken is 20 seconds.
So A catches up with B when $t = 60$.

[2]

5. (a) $u = 7$, $v = 21$, $t = 8$, $s = ?$

$$s = \tfrac{1}{2}(u+v)t$$
$$= \tfrac{1}{2}(7+21)\times 8$$
$$= 112$$

The particle travels 112 m.

[2]

(b) (i) From A to B: $u = u$, $a = a$, $t = 5$, $s = 75$

$$s = ut + \tfrac{1}{2}at^2$$
$$75 = 5u + \tfrac{1}{2}a\times 25$$
$$75 = 5u + 12.5a \quad ①$$

[3]

(ii) From A to C: $u = u$, $a = a$, $t = 15$, $s = 315$

$$s = ut + \tfrac{1}{2}at^2$$
$$315 = 15u + \tfrac{1}{2}a\times 225$$
$$105 = 5u + 37.5a \quad ②$$

[4]

(iii) ② - ①: $30 = 25a$

$$a = 1.2$$

Substituting into ①: $75 = 5u + 12.5 \times 1.2$

$u = 12$

[3]

(iv) From A to D:

$u = 12$
$s = 515$
$a = 1.2$
$v = ?$

$$v^2 = u^2 + 2as$$
$$= 12^2 + 2 \times 1.2 \times 515$$
$$= 1380$$
$$v = 37.1$$

The calculated value of v is quite close to the observed value of 36.5, so the model would seem to be reasonable.

[3]

6. (a)

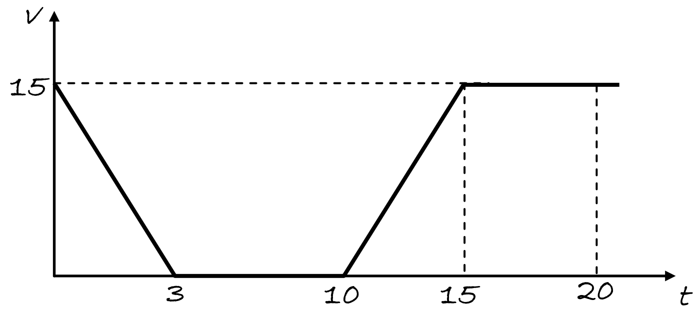
$u = 15$
$v = 0$
$s = 22.5$
$t = ?$

$$s = \tfrac{1}{2}(u+v)t$$
$$22.5 = \tfrac{1}{2}(15+0)t$$
$$45 = 15t$$
$$t = 3$$

so the deceleration takes 3 seconds.

[2]

(b) (i)



Distance travelled by car $= \tfrac{1}{2} \times 15 \times 3 + \tfrac{1}{2} \times 15 \times 5$
$= 22.5 + 37.5$
$= 60$

Distance travelled = 60 m.

[6]

(ii) Distance travelled by bus $= 20 \times 15 = 300$
Extra distance travelled by bus $= 300 - 60 = 240$ m.

[2]

(iii) For the motion after the car reaches its constant speed and the bus begins to decelerate, until the car catches up with the bus:

Let distance travelled by car be d.

Bus:

$u = 20$
$a = -0.2$
$t = T$
$s = d - 240$

$$d - 240 = ut + \tfrac{1}{2}at^2$$
$$d = 240 + 20T - 0.1T^2$$

so the distance travelled by the car is $240 + 20T - 0.1T^2$ m.

Car: $d = 15T$

$240 + 20T - 0.1T^2 = 15T$

$0.1T^2 - 5T - 240 = 0$

$T^2 - 50T - 2400 = 0$

$(T - 80)(T + 30) = 0$

$T = 80$ or $T = -30$

Since $T > 0$, $T = 80$

Bus: $v = u + at$

$= 20 - 0.2 \times 80$

$= 20 - 16$

$= 4$

The bus is travelling at 4 ms^{-1} when the car catches it up.

[7]