

#### **Topic assessment**

1. A practical music examination can be taken once or twice. Those candidates who fail it on the first occasion take it a second time.

For those having their first attempt, 25% pass with distinction and 45% gain an ordinary pass. For those taking the examination for a second time, the corresponding figures are 5% and 70% respectively. The tree diagram below illustrates the situation.



- (i) Find the probability that a randomly chosen candidate(A) fails the examination
  - (B) passes the examination (with or without distinction)
- (ii) Jill and Jo are two randomly chosen entrants for the examination. Find the probabilities that
  - (A) both pass (with or without distinction), but just one of them needs a second attempt,
  - (B) Jill gets a better result than Jo.

[8]

[4]

2. Midchester Rovers and Southpool United are two teams in a football league. Previous experience shows that, when these two teams play each other, the numbers of goals scored are independent and have the following probabilities.

Number of goals	0	1	2	3
Midchester Rovers	0.2	0.3	0.35	0.15
Southpool United	0.4	0.5	0.1	0

The two teams play each other.

Find the probabilities of the events in parts (i) to (v).

- (i) Midchester win the game 1-0. [1]
- (ii) One of the teams wins the game 2-0.



[2]

(iii)	The game ends in a draw.	[3]
(iv)	Southpool win the game.	[3]

The probability that Southpool fail to score in k successive games against Midchester is greater than 1%.

- (v) Find the greatest possible value of *k*.
- 3. In the sixth form at Eastport School, there are 45 students studying at least one of the three sciences, Biology, Chemistry and Physics.

The diagram below shows the number of students studying each science.



- (i) Find the probability that one of these students, selected at random, is studying
  - (A) Physics
  - (B) Chemistry
  - (C) both Physics and Chemistry.

[3]

[4]

[4]

- (ii) Two students are selected at random. Find the probability that both are studying just one science and that they are not both studying the same science.
- 4. A survey is conducted to find which type of property people live in and whether the property is owned or rented by its occupier. The results for a particular region of the country are as follows.

Type of property	Proportion	Proportion of properties	
	of each type	Owned	Rented
Detached / semi-detached	45%	75%	25%
Terraced house	35%	50%	50%
Flat / bedsit	20%	35%	65%

A property is chosen at random.

<ul> <li>(i) Construct a tree diagram to represent the information in the table.</li> <li>(ii) Find the probability that the property is owned.</li> <li>(iii) Find the probability that the property is a terraced house or is rented.</li> </ul>	[3] [3] [3]
Two properties are now chosen at random. (iv) Find the probability that they are of different types.	[4]

5. The probability distribution of a discrete random variable *X* is given by:

(i)

P(X = r) = 
$$k(6r^2 - r^3)$$
 for r = 1, 2, 3, 4, 5  
P(X = r) = 0 otherwise  
(i) Show that  $k = \frac{1}{105}$ . [3]  
(ii) Find the value of P(2 ≤ X < 4). [2]

Total 50 marks

## Solutions to topic assessment

1.



[8]

2. (i) P(Midchester score 1 goal and Southpool score 0) = 
$$0.3 \times 0.4 = 0.12$$
 or  $\frac{3}{25}$   
[1]  
(ii) P(Midchester score 2 and Southpool score 0) =  $0.35 \times 0.4 = 0.14$   
P(Midchester score 0 and Southpool score 2) =  $0.2 \times 0.1 = 0.02$   
P(one team wins  $2 - 0$ ) =  $0.14 + 0.02 = 0.16$  or  $\frac{4}{25}$   
[2]  
(iii) P(score is  $0 - 0$ ) =  $0.2 \times 0.4 = 0.08$   
P(score is  $1 - 1$ ) =  $0.3 \times 0.5 = 0.15$   
P(score is  $2 - 2$ ) =  $0.35 \times 0.1 = 0.035$   
P(score is  $3 - 3$ ) =  $0.15 \times 0 = 0$   
P(score is  $a \, draw$ ) =  $0.08 + 0.15 + 0.035 = 0.265$  or  $\frac{53}{200}$   
[3]  
(iv) P(score is  $0 - 1$ ) =  $0.2 \times 0.5 = 0.1$   
P(score is  $1 - 2$ ) =  $0.3 \times 0.1 = 0.02$   
P(score is  $1 - 2$ ) =  $0.3 \times 0.1 = 0.03$   
P(Southpool win) =  $0.1 + 0.02 + 0.03 = 0.15$  or  $\frac{3}{20}$   
[3]  
(v) P(Southpool fail to score in k successive games) =  $0.4^{k}$   
 $0.4^{k} > 0.01$   
 $0.4^{s} = 0.01024$   
 $0.4^{e} = 0.004096$   
so the largest possible value of k is 5.  
[4]

3. (i) (A) Total number of studying Physics = 4+3+6+3 = 16P(Physics) =  $\frac{16}{45}$ 

> (B) Total number studying Chemistry = 15+2+4+6 = 27P(Chemistry) =  $\frac{27}{45} = \frac{3}{5}$

(C) Total number studying both Physics and Chemistry = 4+6 = 10P(Physics and Chemistry) =  $\frac{10}{45} = \frac{2}{9}$ 

[3]

(ii) 
$$P(P, C) = P(C, P) = \frac{3}{45} \times \frac{2}{44} = \frac{6}{1980}$$
  
 $P(P, B) = P(B, P) = \frac{3}{45} \times \frac{12}{44} = \frac{36}{1980}$   
 $P(C, B) = P(B, C) = \frac{2}{45} \times \frac{12}{44} = \frac{24}{1980}$ 

Probability = 
$$2 \times \frac{6+36+24}{1980} = \frac{66}{990} = \frac{1}{15} (= 0.0667 \text{ to } 3 \text{ s.f.})$$
[4]

4. (í)



(ii)  $P(owned) = (0.45 \times 0.75) + (0.35 \times 0.5) + (0.2 \times 0.35)$ = 0.5825 or  $\frac{233}{400}$ 

- (iii) P(terraced house or rented) =  $(0.45 \times 0.25) + 0.35 + (0.2 \times 0.65)$ = 0.5925 or  $\frac{237}{400}$
- (iv) Probability that they are of the same type =  $0.45^2 + 0.35^2 + 0.2^2$ Probability that they are of different types =  $1 - (0.45^2 + 0.35^2 + 0.2^2)$ = 0.635 or  $\frac{127}{200}$

[4]

[3]

[3]

[3]

[3]

5. (í)

<b>v</b> · <i>y</i>						
	r	1	2	3	4	5
	P(X = r)	5k	16k	27k	32k	25k

5k + 16k + 27k + 32k + 25k = 1105k = 1  $k = \frac{1}{105}$ 

(ii) 
$$P(2 \le X < 4) = \frac{16}{105} + \frac{27}{105} = \frac{43}{105} = 0.410 \text{ (3 s.f.)}$$
 [2]