

**Topic assessment**

1. Sweets called “Scruffies” are sold in packets of 18.

Scruffies come in a variety of colours, and market research shows that red is the most popular. Scruffies are packed randomly, and on average 25% are red.

- (i) Find the probability that a packet contains no more than 6 red Scruffies. [2]  
(ii) Find the probability that a packet contains exactly 4 red Scruffies. [2]

Because of a fault in the packing machine, one packet in ten now contains 19 Scruffies instead of 18.

- (iii) Find the new probability that a randomly selected packet contains exactly 4 red Scruffies. [5]

In order to increase sales, the manufacturer claims to have increased the proportion of red Scruffies. Eric decides to test the manufacturer’s claim by purchasing a packet of Scruffies. (Assume that all packets now contain 18 Scruffies.)

- (iv) The packet contains 8 red Scruffies. Does this support the manufacturer’s claim at the 5% significance level? Write down your hypotheses clearly and show your working in full. [6]

2. In a national survey, it was found that 40% of young people failed a standard fitness test.

A random sample of 30 young people is taken.

- (i) Explain why, for such random samples, the mean number of those who failed the fitness test is 12. [1]  
(ii) Find the probability of the mean number occurring. Explain how you would check whether the mean is also the modal value. *No further calculations are necessary.* [4]

A local health authority wishes to investigate whether or not the proportion of young people in its area who failed the fitness test is 40%. A random sample of 20 young people contains 12 who failed the test.

- (iii) Carry out a suitable hypothesis test at the 5% significance level, stating your hypothesis and conclusions carefully. Find the critical region for the test. [10]

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3. *Joggers* produce packets of crisps. On average, 1 in every 5 packets, chosen randomly, contains a prize voucher.

A box contains 30 packets of *Joggers* crisps.

- (i) State the expected number of packets containing a prize voucher and find the probability of exactly this number occurring. [4]
- (ii) Show that it is almost certain that at least one packet will contain a voucher. [2]

*Sprinters* also produce packets of crisps, some of which contain a prize voucher. Jean wishes to test whether the proportion of packets of *Sprinters* crisps with prize vouchers is also  $\frac{1}{5}$ .

- (iii) State suitable null and alternative hypotheses for the test. [2]

Jean buys 12 packets of *Sprinters* crisps and finds no vouchers at all.

- (iv) Carry out the hypothesis test at the 5% significance level, giving the critical region for the test and stating your conclusions carefully. [5]
- (v) How many packets of crisps would Jean have to buy for the critical region to have a non-empty lower tail? [2]
4. A police road-safety team examines the tyres of a large number of commercial vehicles. They find that 17% of lorries and 20% of vans have defective tyres.

- (i) Six lorries are stopped at random by the road-safety team. Find the probability that
- (A) none of the lorries have defective tyres, [2]
- (B) exactly two lorries have defective tyres. [3]

Following a road-safety campaign to reduce the proportion of vehicles with defective tyres, 18 vans are stopped at random and their tyres are inspected. Just one of the vans has defective tyres.

You are to carry out a suitable hypothesis test to examine whether the campaign appears to have been successful.

- (ii) State your hypotheses clearly, justifying the form of the alternative hypothesis. [3]
- (iii) Carry out the test at the 5% significance level, stating your conclusions clearly. [3]
- (iv) State, with a reason, the critical value for the test. [2]
- (v) Give a level of significance such that you would come to the opposite conclusion for your test. Explain your reasoning. [2]

**Total 60 marks**

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## Solutions to topic assessment

1. (i) Let  $X$  be the number of red Scruffies in a packet.  
 $X \sim B(18, 0.25)$

From tables,  $P(X \leq 6) = 0.8610$

[2]

$$\begin{aligned} \text{(ii)} \quad P(X = 4) &= {}^{18}C_4 (0.25)^4 (0.75)^{14} \\ &= \frac{18 \times 17 \times 16 \times 15}{1 \times 2 \times 3 \times 4} (0.25)^4 (0.75)^{14} \\ &= 0.213 \text{ (3 s.f.)} \end{aligned}$$

Alternatively, use tables:  $P(X = 4) = P(X \leq 4) - P(X \leq 3)$

$$\begin{aligned} &= 0.5187 - 0.3057 \\ &= 0.2130 \end{aligned}$$

[2]

$$\begin{aligned} \text{(iii)} \quad P(18 \text{ Scruffies and } X = 4) &= 0.9 \times {}^{18}C_4 (0.25)^4 (0.75)^{14} \\ &= 0.9 \times \frac{18 \times 17 \times 16 \times 15}{1 \times 2 \times 3 \times 4} (0.25)^4 (0.75)^{14} \\ &= 0.1917 \end{aligned}$$

$$\begin{aligned} P(19 \text{ Scruffies and } X = 4) &= 0.1 \times {}^{19}C_4 (0.25)^4 (0.75)^{15} \\ &= 0.1 \times \frac{19 \times 18 \times 17 \times 16}{1 \times 2 \times 3 \times 4} (0.25)^4 (0.75)^{15} \\ &= 0.0202 \end{aligned}$$

$$P(X = 4) = 0.1917 + 0.0202 = 0.212 \text{ (3 s.f.)}$$

[5]

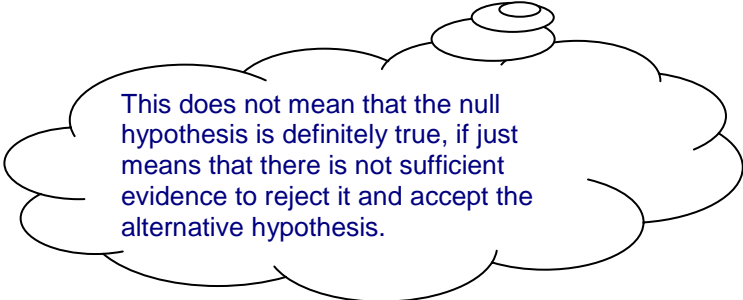
- (iv) Let  $p$  be the probability that a Scruffie is red  
 $H_0: p = 0.25$   
 $H_1: p > 0.25$   
Significance level = 5%

$$\begin{aligned} \text{For } B(18, 0.25), P(X \geq 8) &= 1 - P(X \leq 7) \\ &= 1 - 0.9431 \\ &= 0.0569 \end{aligned}$$

Probability  $> 0.05$ , so accept  $H_0$ .

There is not sufficient evidence to support the manufacturer's claim to have increased the proportion of red Scruffies.

[6]



This does not mean that the null hypothesis is definitely true, it just means that there is not sufficient evidence to reject it and accept the alternative hypothesis.

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2. (i) This is a binomial distribution  $B(30, 0.4)$   
For the binomial distribution, mean =  $np = 30 \times 0.4 = 12$

[1]

- (ii) Let  $X$  be the number of young people in a sample of 30 who fail the test.

$$\begin{aligned}P(X = 12) &= {}^{30}C_{12} (0.4)^{12} (0.6)^{18} \\ &= \frac{30!}{12!18!} (0.4)^{12} (0.6)^{18} \\ &= 0.147 \text{ (3 s.f.)}\end{aligned}$$

In general, to check whether the mean is also the modal value, you would calculate the probabilities of the values either side (in this case  $P(X = 11)$  and  $P(X = 13)$ ) and see if these are larger or smaller. However, in this case since the mean is an integer, then the mode must be equal to the mean.

[4]

- (iii) Let  $p$  be the probability that a young person fails the test.

$$H_0: p = 0.4$$

$$H_1: p \neq 0.4$$

Significance level = 5%

12 young people failed, which is in the upper tail.

$$\begin{aligned}\text{For } B(30, 0.4), P(X \geq 12) &= 1 - P(X \leq 11) \\ &= 1 - 0.9435 \\ &= 0.0565\end{aligned}$$

$0.0565 > 0.025$ , so accept  $H_0$ .

There is not sufficient evidence to suggest that the proportion of young people in this area who failed the test is different from 40%.

For the lower tail critical region, need the highest value of  $a$  such that  $P(X \leq a) < 0.025$ .

$$\text{From tables, } P(X \leq 3) = 0.0160$$

$$P(X \leq 4) = 0.0510$$

so the highest value of  $a$  is 3.

For the upper tail critical region, need the lowest value of  $b$  such that

$$P(X \geq b) < 0.025 \Rightarrow 1 - P(X \leq b - 1) < 0.025$$

$$\Rightarrow P(X \leq b - 1) > 0.975$$

$$\text{From tables, } P(X \leq 11) = 0.9435$$

$$P(X \leq 12) = 0.9790$$

The lowest value of  $b - 1$  is 12, so the lowest value of  $b$  is 13.

The critical region is  $X \leq 3$  or  $X \geq 13$ .

[10]

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3. (i) Expected number =  $30 \times \frac{1}{5} = 6$

Let  $X$  be the number of packets which contain a voucher.

$$\begin{aligned}P(X = 6) &= {}^{30}C_6 \left(\frac{1}{5}\right)^6 \left(\frac{4}{5}\right)^{24} \\ &= \frac{30!}{6!24!} \left(\frac{1}{5}\right)^6 \left(\frac{4}{5}\right)^{24} \\ &= 0.179 \text{ (3 s.f.)}\end{aligned}$$

[4]

(ii)  $P(X \geq 1) = 1 - P(X = 0)$

$$\begin{aligned}&= 1 - \left(\frac{4}{5}\right)^{30} \\ &= 0.999 \text{ (3 s.f.)}\end{aligned}$$

This probability is very nearly 1, so it is almost certain that at least one packet contains a voucher.

[2]

(iii) Let  $p$  be the probability that a packet contains a voucher.

$$H_0: p = \frac{1}{5}$$

$$H_1: p \neq \frac{1}{5}$$

Significance level = 5%

[2]

(iv) Let  $Y$  be the number of packets containing a voucher.

$$Y \sim B(12, 0.2)$$

For the lower tail, need the highest possible value of  $a$  for which  $P(Y \leq a) < 0.025$ .

From tables,  $P(Y \leq 0) = 0.0687$   
so the lower tail is empty.

For the upper tail, need the lowest possible value of  $b$  for which

$$\begin{aligned}P(Y \geq b) < 0.025 &\Rightarrow 1 - P(Y \leq b-1) < 0.025 \\ &\Rightarrow P(Y \leq b-1) > 0.975\end{aligned}$$

$$\begin{aligned}\text{From tables, } P(Y \leq 4) &= 0.9274 \\ P(Y \leq 5) &= 0.9806\end{aligned}$$

The lowest possible value of  $b-1$  is 5, so the lowest possible value of  $b$  is 6.

The critical region is  $Y \geq 6$ .

$Y = 0$  which does not lie in the critical region, so accept  $H_0$ . There is not sufficient evidence to suggest that the proportion of packets containing

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vouchers is different from  $\frac{1}{5}$ .

[5]

(v) For the lower tail to be non-empty,  $P(Y = 0) < 0.025$

$$\Rightarrow (0.8)^n > 0.025$$

$$(0.8)^{16} = 0.028$$

$$(0.8)^{17} = 0.023$$

so she would need to buy at least 17 packets of crisps.

[2]

4. (i) Let  $X$  be the number of lorries with defective tyres

$$X \sim B(6, 0.17)$$

$$(A) P(X = 0) = (0.83)^6 = 0.327 \text{ (3 s.f.)}$$

$$\begin{aligned} (B) P(X = 2) &= {}^6C_2 (0.17)^2 (0.83)^4 \\ &= \frac{6 \times 5}{1 \times 2} (0.17)^2 (0.83)^4 \\ &= 0.206 \text{ (3 s.f.)} \end{aligned}$$

[2]

(ii) Let  $p$  be the probability that a lorry has defective tyres

$$H_0: p = 0.2$$

$$H_1: p < 0.2$$

Significance level = 5%

The alternative hypothesis takes this form because we are looking only at a possible decrease in the proportion of vehicles with defective tyres.

[3]

(iii) Let  $Y$  be the number of vans with defective tyres

$$Y \sim B(18, 0.2)$$

$$P(Y \leq 1) = 0.0991$$

Probability  $> 0.05$ , so accept  $H_0$ . There is not sufficient evidence to suggest that there has been a reduction in the proportion of defective tyres.

[3]

(iv) The critical value is 0, since  $P(Y = 0) < 0.05$ .

[2]

(v) For  $H_0$  to be rejected,  $P(Y \leq 1)$  must be less than the significance level. So the significance level must be above 9.91%. So at the 10% level,  $H_0$  would be rejected.

[2]