EdExcel AS Mathematics Vectors

Topic assessment

1. Given that $\mathbf{p} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

   (i) Find $2\mathbf{p} + \mathbf{q}$. [2]

   (ii) Find a and b such that $a\mathbf{p} + b\mathbf{q} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. [4]

2. The points A, B and C have coordinates (2, -1), (3, 2) and (-4, 0) respectively.

   (i) Write down the vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{BC}$. [3]

   (ii) Find $\left|\overrightarrow{BC}\right|$. [2]

   (iii) Find a unit vector in the direction of $\overrightarrow{AB}$. [2]

3. The points P, Q and R have coordinates (1, 4), (2, 1) and (-2, 3) respectively.

   (i) The point S is such that $\overrightarrow{SR} = \overrightarrow{PQ}$. Find the coordinates of S. [4]

   (ii) What shape is quadrilateral PQRS? [1]

   (iii) M is the midpoint of PR. Find the vector $\overrightarrow{QM}$. [2]

4. Three forces $\mathbf{F}_1 = 2\mathbf{i} + 3\mathbf{j}$ N, $\mathbf{F}_2 = 5\mathbf{i} - \mathbf{j}$ N and $\mathbf{F}_3 = 3\mathbf{i} + a\mathbf{j}$ N, act on a box.
   The resultant force acts in the direction $2\mathbf{i} + \mathbf{j}$.
   Find the value of a and the magnitude of the resultant force. [5]

Total 25 marks

EdExcel AS Maths Vectors Assessment solutions

Solutions to topic assessment

1. (i) $2\begin{pmatrix}1\\4\end{pmatrix}+\begin{pmatrix}3\\-1\end{pmatrix}=\begin{pmatrix}2\\8\end{pmatrix}+\begin{pmatrix}3\\-1\end{pmatrix}=\begin{pmatrix}5\\7\end{pmatrix}$ [2]

(ii) $a\begin{pmatrix}1\\4\end{pmatrix}+b\begin{pmatrix}3\\-1\end{pmatrix}=\begin{pmatrix}1\\0\end{pmatrix} \Rightarrow \begin{matrix}a+3b=1\\4a-b=0\end{matrix}$

$b=4a$

$a+12a=1$

$a=\frac{1}{13}, b=\frac{4}{13}$

[4]

2. (i) $\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=\begin{pmatrix}3\\2\end{pmatrix}-\begin{pmatrix}2\\-1\end{pmatrix}=\begin{pmatrix}1\\3\end{pmatrix}$

$\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}=\begin{pmatrix}-4\\0\end{pmatrix}-\begin{pmatrix}2\\-1\end{pmatrix}=\begin{pmatrix}-6\\1\end{pmatrix}$

$\overrightarrow{BC}=\overrightarrow{OC}-\overrightarrow{OB}=\begin{pmatrix}-4\\0\end{pmatrix}-\begin{pmatrix}3\\2\end{pmatrix}=\begin{pmatrix}-7\\-2\end{pmatrix}$ [3]

(ii) $\left|\overrightarrow{BC}\right|=\sqrt{(-7)^2+(-2)^2}=\sqrt{49+4}=\sqrt{53}$ [2]

(iii) $\left|\overrightarrow{AB}\right|=\sqrt{1^2+3^2}=\sqrt{10}$

Unit vector is $\frac{1}{\sqrt{10}}\begin{pmatrix}1\\3\end{pmatrix}$ [2]

3. (i) $\overrightarrow{SR}=\overrightarrow{PQ}$

$\overrightarrow{OR}-\overrightarrow{OS}=\overrightarrow{OQ}-\overrightarrow{OP}$

$\begin{pmatrix}-2\\3\end{pmatrix}-\overrightarrow{OS}=\begin{pmatrix}2\\1\end{pmatrix}-\begin{pmatrix}1\\4\end{pmatrix}$

$\overrightarrow{OS}=\begin{pmatrix}-2\\3\end{pmatrix}-\begin{pmatrix}2\\1\end{pmatrix}+\begin{pmatrix}1\\4\end{pmatrix}=\begin{pmatrix}-3\\6\end{pmatrix}$

[4]

(ii) Parallelogram [1]

(iii) M has coordinates $\left(\frac{1-2}{2}, \frac{4+3}{2}\right) = \left(-\frac{1}{2}, \frac{7}{2}\right)$

$$\overrightarrow{QM} = \overrightarrow{OM} - \overrightarrow{OQ}$$

$$= \begin{pmatrix} -\frac{1}{2} \\ \frac{7}{2} \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{5}{2} \\ \frac{5}{2} \end{pmatrix}$$

[2]

4. Resultant force $= 10\underline{i} + (2+a)\underline{j}$

Since resultant force acts in the direction of $2\underline{i} + \underline{j}$, $10 = 2(2+a)$

$5 = 2 + a$

$a = 3$

Resultant force $= 10\underline{i} + 5\underline{j}$

Magnitude of force $= \sqrt{10^2 + 5^2} = \sqrt{125} = 5\sqrt{5}$ N. [5]