Topic assessment

1. Solve each of the following quadratic equations, if possible, giving answers in exact form.
   (i) $2x^2 - x - 3 = 0$
   (ii) $3x^2 - 2x + 4 = 0$
   (iii) $x^2 + 5x - 1 = 0$ [5]

2. (i) Write the quadratic expression $x^2 + 4x + 5$ in the form $A(x+B)^2 + C$. [2]
   (ii) Find the discriminant of the quadratic equation $x^2 + 4x + 5 = 0$. [2]
   (iii) What does the value of this discriminant tell you about the roots of the equation $x^2 + 4x + 5 = 0$? [1]
   (iv) Sketch the graph of $y = x^2 + 4x + 5$, showing the coordinates of the turning point and any points where the curve crosses the coordinate axes. [3]

3. (i) By factorising, solve the equation $2x^2 + x - 6 = 0$. [2]
   (ii) Sketch the graph of $y = 2x^2 + x - 6$, showing the coordinates of any points where the graph cuts the coordinate axes. [3]

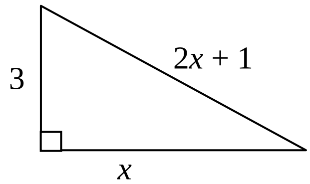
4. The quadratic equation $2x^2 + 5x + k = 0$ has equal roots.
   (i) Find the value of *k*. [3]
   (ii) Solve the equation $2x^2 + 5x + k = 0$. [2]

5. (i) Write the expression $2x^2 + 2x - 1$ in the form $a(x+p)^2 + q$. [3]
   (ii) Hence, or otherwise, solve the equation $2x^2 + 2x - 1 = 0$. [2]

6. Sketch the graph of $y = 12 + 4x - x^2$, showing the coordinates of any points where the graph cuts the coordinate axes. [4]

7. Solve these equations, giving your answers in exact form.
   (i) $x^{\frac{2}{3}} + x^{\frac{1}{3}} - 6 = 0$ [4]
   (ii) $x^4 + 3x^2 - 10 = 0$ [4]

8. The diagram shows a right-angled triangle. Find the value of *x*, correct to 3 s.f. [4]



9. Amy throws a ball so that when it is at its highest point, it passes through a hoop. The path of the ball is modelled by the equation $y = h + kx - \frac{1}{2}x^2$, where *y* is the height of the ball above the ground and *x* is the horizontal distance from the point at which the ball was thrown. The centre of the hoop is at the point where $x = 2$ and $y = 5$.
   Find the values of *h* and *k*, and find the value of *x* at which the ball hits the ground. [6]

Total 50 marks

Edexcel AS Maths Quadratics Assessment solutions

Solutions to topic assessment

1. (i) $2x^2 - x - 3 = 0$

$a = 2, b = -1, c = -3$

Discriminant $= b^2 - 4ac = (-1)^2 - 4 \times 2 \times -3 = 1 + 24 = 25$

Since the discriminant is a perfect square, the equation can be factorised.

$(2x-3)(x+1) = 0$

$x = \frac{3}{2}$ or $x = -1$

[2]

(ii) $3x^2 - 2x + 4 = 0$

$a = 3, b = -2, c = 4$

Discriminant $= b^2 - 4ac = (-2)^2 - 4 \times 3 \times 4 = 4 - 48 = -44$

The discriminant is negative, so the equation has no real roots.

[1]

(iii) $x^2 + 5x - 1 = 0$

$a = 1, b = 5, c = -1$

Discriminant $= b^2 - 4ac = 5^2 - 4 \times 1 \times -1 = 25 + 4 = 29$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{29}}{2 \times 1} = \frac{-5 \pm \sqrt{29}}{2}$$

[2]

2. (i) $x^2 + 4x + 5 = (x+2)^2 - 4 + 5$

$= (x+2)^2 + 1$

[2]

(ii) $x^2 + 4x + 5 = 0$

$a = 1, b = 4, c = 5$

Discriminant $= b^2 - 4ac = 4^2 - 4 \times 1 \times 5 = 16 - 20 = -4$

[2]

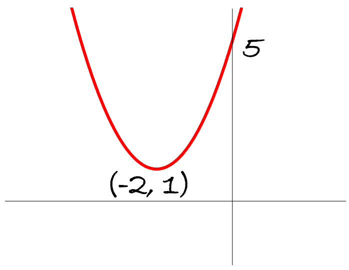
(iii) Since the discriminant is negative, there are no real solutions to to the equation $x^2 + 4x + 5 = 0$.

[1]

(iv) The graph of $y = x^2 + 4x + 5$ cuts the y-axis at (0, 5).

It does not cut the x-axis.

It has line of symmetry x = -2.



[3]

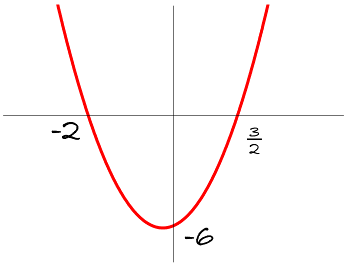
3. (i) $2x^2 + x - 6 = 0$

$(2x-3)(x+2) = 0$

$x = \frac{3}{2}$ or $x = -2$

[2]

(ii) The graph cuts the x-axis at $\left(\frac{3}{2}, 0\right)$ and $(-2, 0)$, and cuts the y-axis at $(0, -6)$.



[3]

4. (i) $2x^2 + 5x + k = 0$

$a = 2, b = 5, c = k$

If roots are equal, $b^2 - 4ac = 0$

$5^2 - 4 \times 2 \times k = 0$

$8k = 25$

$k = \dfrac{25}{8}$

[3]

(ii) $2x^2 + 5x + \frac{25}{8} = 0$

$16x^2 + 40x + 25 = 0$

$(4x+5)^2 = 0$

$x = -\frac{5}{4}$

[2]

5. (i) $2x^2 + 2x - 1 = 2\left(x^2 + x\right) - 1$

$= 2\left(\left(x + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2\right) - 1$

$= 2\left(x + \frac{1}{2}\right)^2 - 2 \times \frac{1}{4} - 1$

$= 2\left(x + \frac{1}{2}\right)^2 - \frac{1}{2} - 1$

$= 2\left(x + \frac{1}{2}\right)^2 - \frac{3}{2}$

[3]

(ii) $2x^2+2x-1=0$

$2\left(x+\frac{1}{2}\right)^2-\frac{3}{2}=0$

$2\left(x+\frac{1}{2}\right)^2=\frac{3}{2}$

$\left(x+\frac{1}{2}\right)^2=\frac{3}{4}$

$x+\frac{1}{2}=\pm\frac{\sqrt{3}}{2}$

$x=-\frac{1}{2}\pm\frac{\sqrt{3}}{2}$

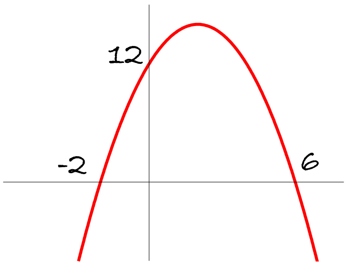
[2]

6. $y=12+4x-x^2$

$=(6-x)(2+x)$

When $x=0$, $y=12$

When $y=0$, $x=6$ or -2



[4]

7. (i) $x^{\frac{2}{3}}+x^{\frac{1}{3}}-6=0$

Let $y=x^{\frac{1}{3}}$

$y^2+y-6=0$

$(y+3)(y-2)=0$

$y=-3$ or 2

$x=y^3 \Rightarrow x=-27$ or 8

[4]

(ii) $x^4+3x^2-10=0$

Let $y=x^2$

$y^2+3y-10=0$

$(y+5)(y-2)=0$

$y=-5$ or 2

$x=\sqrt{y} \Rightarrow x=\pm\sqrt{2}$

[4]

8. Using Pythagoras' theorem: $(2x+1)^2 = x^2 + 3^2$

$$4x^2 + 4x + 1 = x^2 + 9$$
$$3x^2 + 4x - 8 = 0$$
$$x = \frac{-4 \pm \sqrt{4^2 - 4\times 3\times -8}}{2\times 3} = 1.10 \text{ or } -2.43$$

Since x is a length, it must be positive, so $x = 1.10$ (3 s.f.)

[4]

9. The highest point is (2, 5) so the equation of the path is of the form

$$y = 5 - a(x-2)^2$$
$$= 5 - ax^2 + 4ax - 4a$$
$$= 5 - 4a + 4ax - ax^2$$

Comparing with $y = h + kx - \frac{1}{2}x^2$ gives $a = \frac{1}{2}$

$$k = 4a = 2$$
$$h = 5 - 4a = 5 - 2 = 3$$

When the ball hits the ground, $3 + 2x - \frac{1}{2}x^2 = 0$

$$x^2 - 4x - 6 = 0$$
$$x = \frac{4 \pm \sqrt{4^2 - 4\times 1\times -6}}{2} = 5.16 \text{ or } -2.32$$

The value of x must be positive, so $x = 5.16$.