Edexcel AS Mathematics Polynomials

Topic assessment

1. (i) Add (x^3+2x^2-3x+1) to $(2x^3+5x-3)$ [2]
   (ii) Subtract $2x^3-3x^2+x-2$ from $(x^4+x^3-2x^2+1)$ [2]
   (iii) Multiply (x^3+4x^2-2x+3) by $(2x-1)$ [3]
   (iv) Multiply (x^2+2x+3) by (x^2-x+1) [3]
   (v) Divide $(2x^3-x^2+3x-4)$ by $(x-1)$ [3]

2. $(x-3)$ is a factor of the polynomial x^3+ax^2-5x+6.
   Find the value of a. [2]

3. (i) Solve the equation $2x^3-x^2-5x-2=0$. [4]
   (ii) Sketch the graph of $y=2x^3-x^2-5x-2$. [3]

4. (i) Show that $(x-3)$ is a factor of $6x^3-17x^2-5x+6$. [1]
   (ii) Hence solve the equation $6x^3-17x^2-5x+6=0$. [2]
   (iii) Sketch the graph of $y=6x^3-17x^2-5x+6$. [3]

5. $\mathrm{f}(x)=x^3+ax^2+bx+8$.
   (i) $(x-1)$ and $(x-2)$ are factors of $\mathrm{f}(x)$.
   Find the values of a and b. [4]
   (ii) Factorise $\mathrm{f}(x)$ completely and hence solve the equation $\mathrm{f}(x)=0$. [3]
   (ii) Sketch the graph of $y=\mathrm{f}(x)$. [3]

6. (i) Sketch the curve $y=(2x+1)(x-2)^2$.
   Draw the line $y=x+2$ on your graph and show that it intersects with the curve at the point $x=1$. [5]
   (iii) Show that the x-coordinates of the points where the line and the curve intersect satisfy the equation $2x^3-7x^2+3x+2=0$. [3]
   (iv) Find the x-coordinates of the other two points of intersection of the line and the curve, giving your answers to 2 decimal places. [4]

Total 50 marks

Edexcel AS Maths Polynomials Assessment solutions

Solutions to topic assessment

1. (i)

$$\begin{array}{rrrr} x^3 & +2x^2 & -3x & +1 \\ 2x^3 & & +5x & -3 \\ \hline 3x^3 & +2x^2 & +2x & -2 \end{array}$$

[2]

(ii)

$$\begin{array}{rrrrrr} & x^4 & x^3 & -2x^2 & & +1 \\ - & & 2x^3 & -3x^2 & +x & -2 \\ \hline & x^4 & -x^3 & +x^2 & -x & +3 \end{array}$$

[2]

(iii)

$$\begin{array}{rrrrrr} (x^3+4x^2-2x+3)(2x-1) = & 2x^4 & +8x^3 & -4x^2 & +6x & \\ & & -x^3 & -4x^2 & +2x & -3 \\ \hline & 2x^4 & +7x^3 & -8x^2 & +8x & -3 \end{array}$$

[3]

(iv)

$$\begin{array}{rrrrrr} (x^2+2x+3)(x^2-x+1) = & x^4 & +2x^3 & +3x^2 & & \\ & & -x^3 & -2x^2 & -3x & \\ & & & x^2 & +2x & +3 \\ \hline & x^4 & +x^3 & +2x^2 & -x & +3 \end{array}$$

[3]

(v)

$$2x^3 - x^2 + 3x - 4 = (x-1)(2x^2 + x + 4)$$

$$\frac{2x^3 - x^2 + 3x - 4}{x-1} = 2x^2 + x + 4$$

[3]

2. $f(x) = x^3 + ax^2 - 5x + 6$

By the factor theorem, (x – 3) is a factor $\Rightarrow f(3) = 0$

$$\Rightarrow 27 + 9a - 15 + 6 = 0$$

$$\Rightarrow 9a = -18$$

$$\Rightarrow a = -2$$

[2]

3. (i)

$$f(x) = 2x^3 - x^2 - 5x - 2$$

$$f(1) = 2 - 1 - 5 - 2 = -6$$

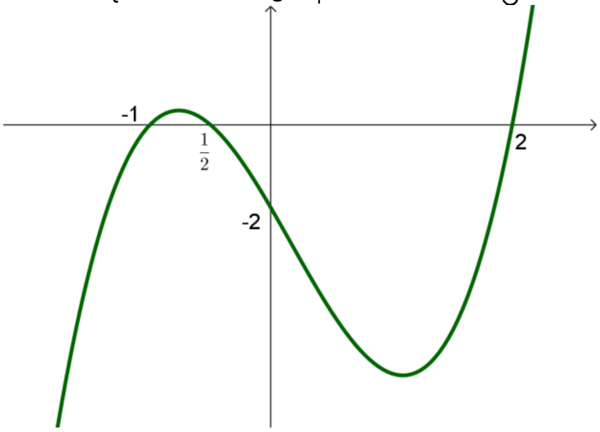
$$f(-1) = -2 - 1 + 5 - 2 = 0$$

so by the factor theorem, (x + 1) is a factor.

$$2x^3 - x^2 - 5x - 2 = 0$$
$$(x+1)(2x^2 - 3x - 2) = 0$$
$$(x+1)(x-2)(2x+1) = 0$$
$$x = -1 \text{ or } x = 2 \text{ or } x = -\tfrac{1}{2}$$

[4]

(ii) From (i), the graph of $y = 2x^3 - x^2 - 5x - 2$ crosses the x-axis at (-1, 0), (2, 0) and $(-\frac{1}{2}, 0)$.
From the equation, the graph crosses the y-axis at (0, -2).



[3]

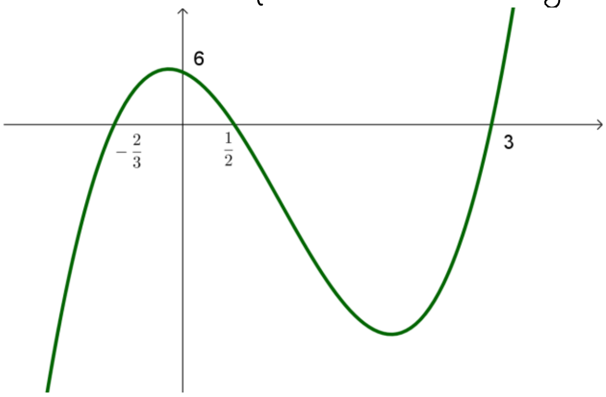
4. (i) $f(x) = 6x^3 - 17x^2 - 5x + 6$
$f(3) = 162 - 153 - 15 + 6 = 0$
so by the factor theorem (x – 3) is a factor.

[1]

(ii) $6x^3 - 17x^2 - 5x + 6 = 0$
$$(x-3)(6x^2 + x - 2) = 0$$
$$(x-3)(2x-1)(3x+2) = 0$$
$$x = 3 \text{ or } x = \tfrac{1}{2} \text{ or } x = -\tfrac{2}{3}$$

[2]

(iii) From (ii), the graph crosses the x-axis at (3, 0), $(\frac{1}{2}, 0)$ and $(-\frac{2}{3}, 0)$.
From the equation, it crosses the y-axis at (0, 6).



[3]

5. (i) $f(x) = x^3 + ax^2 + bx + 8$

$(x-1)$ is a factor $\Rightarrow f(1) = 0$

$\Rightarrow 1 + a + b + 8 = 0$

$\Rightarrow a + b = -9$

$(x-2)$ is a factor $\Rightarrow f(2) = 0$

$\Rightarrow 8 + 4a + 2b + 8 = 0$

$\Rightarrow 2a + b = -8$

Subtracting $\Rightarrow a = 1, b = -10$

[4]

(ii) Two factors are $(x-1)(x-2)$ so $(x^2 - 3x + 2)$ is a quadratic factor.

$f(x) = x^3 + x^2 - 10x + 8$

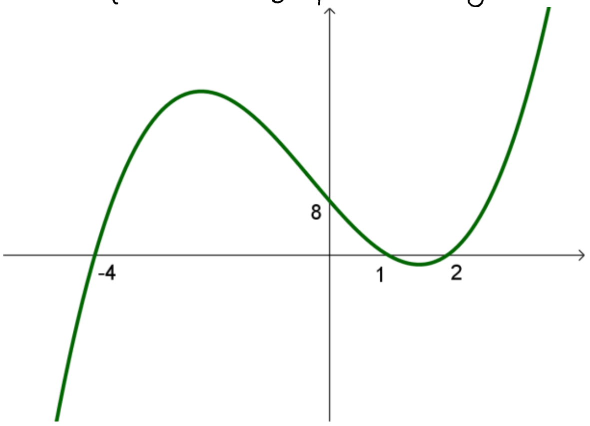
$= (x^2 - 3x + 2)(x + 4)$

$= (x-1)(x-2)(x+4)$

Roots of equation are $x = 1, 2, -4$

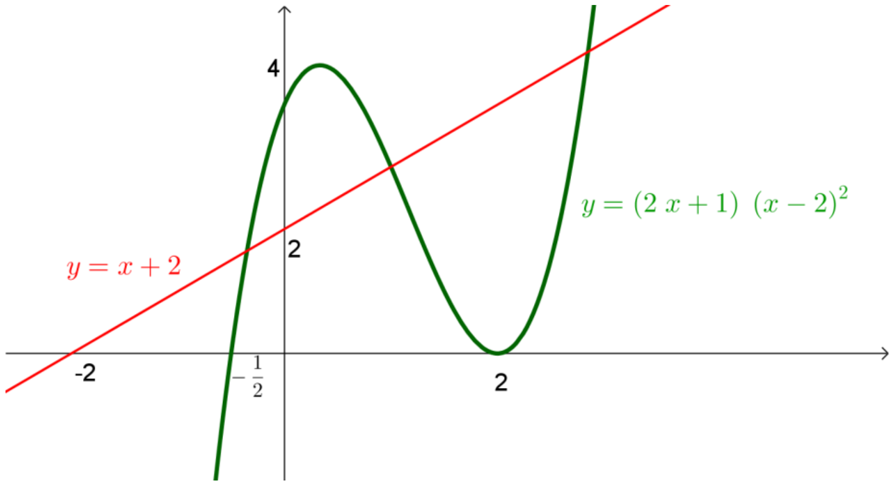
[3]

(iii) From (ii), the graph cuts the x-axis at (2, 0), (1, 0) and (-4, 0).
From the equation, the graph cuts the y-axis at (0, 8).



[3]

6. (i)



When $x = 1$, for the curve $y = (2 \times 1 + 1)(1 - 2)^2 = 3(-1)^2 = 3$
for the line $y = x + 2 = 1 + 2 = 3$.
So both the line and the curve pass through (1, 3) and therefore they intersect when $x = 1$.

[5]

(ii) At intersections, $(2x+1)(x-2)^2 = x+2$

$$(2x+1)(x^2-4x+4) = x+2$$
$$2x^3 + x^2 - 8x^2 - 4x + 8x + 4 = x+2$$
$$2x^3 - 7x^2 + 3x + 2 = 0$$

[3]

(iii) From above, $x = 1$ is a root, so $(x-1)$ is a factor.

$$2x^3 - 7x^2 + 3x + 2 = 0$$
$$(x-1)(2x^2 - 5x - 2) = 0$$

The other two x-coordinates are the roots of $2x^2 - 5x - 2 = 0$

$$x = \frac{5 \pm \sqrt{(-5)^2 - 4 \times 2 \times -2}}{2 \times 2}$$
$$= -0.35 \text{ and } 2.85$$

[4]