Topic assessment

1. Write as a single logarithm:
   (i) $2\log a + 3\log b$ (ii) $\log x - 3\log y + 4\log z$ [4]

2. Express the following in terms of log p, log q and log r.
   (i) $\log \dfrac{pq}{r}$ (ii) $\log \dfrac{\sqrt{p}}{r^2}$ [4]

3. Solve the following equations;
   (i) $2^x = 7$ (ii) $3^{2x} = 5$ [4]

4. Solve the equations
   (i) $2e^x = 3e^{-x} + 5$ [3]
   (ii) $\ln(2x+1) = \ln x + 2$ [3]
   giving your answers in exact form.

5. Alice puts £500 in a savings account, at a fixed interest rate of 5% per year, when her grandson Harry is born. Interest is added to the account on Harry's birthday each year. The amount, P, in the account after n years is given by:
   $$P = 500 \times 1.05^n$$
   How old will Harry be when the amount in the savings account first exceeds £1000? [4]

6. The number N of rabbits in a colony after t years is modelled by $N = 20 \times 2^{0.8t}$.
   (i) How many rabbits are in the colony after 5 years? [2]
   (ii) A biologist suggests that due to limited resources, this model will no longer be appropriate when N reaches 2000. For how many years will this model be appropriate? [3]

7. The temperature T°C of the water in a kettle t minutes after boiling is modelled by the equation $T = 20 + 80e^{-0.5t}$.
   (i) What is the initial temperature of the water? [1]
   (ii) Find the temperature of the water after 5 minutes. [2]
   (iii) Find the time at which the temperature of the water is 30°C. [3]
   (iv) Find the initial rate of cooling, and the rate of cooling after 2 minutes. [3]
   (v) What will be the long-term temperature of the water? [1]

8. In an experiment, the number of bacteria, N, in a culture was estimated at time t days after the measurements started.
   The results were as follows:

t	1	2	3	4	5	6
N	120	170	250	400	620	910

It is believed that the relationship between N and t can be expressed in the form

$$N = ab^t$$

where a and b are constants.

(i) Explain how this can be tested by plotting log N against t. [2]

(ii) Make out a table of values of log N and draw the graph. [3]

(iii) Use your graph to estimate the values of a and b. [3]

(iv) Estimate the number of bacteria present after 20 days. State, with a reason, whether your estimate is likely to be a good one. [2]

9. It is believed that two quantities, x and y, are connected by a relationship of the form $y = kx^n$, where k and n are constants.

In an experiment, the following data were produced.

x	5	10	15	20	25	30	35
y	9	24	48	69	102	131	166

(i) Explain how the form of the relationship can be tested by plotting log y against log x. [2]

(ii) Make out a table of values of log x and log y and plot the graph. [3]

(iii) Use your graph to estimate the values of k and n. [3]

Total 55 marks

Solutions to topic assessment

1. (i) $2\log a + 3\log b = \log a^2 + \log b^3$

$$= \log\left(a^2 b^3\right)$$

[2]

(ii) $\log x - 3\log y + 4\log z = \log x - \log y^3 + \log z^4$

$$= \log \frac{xz^4}{y^3}$$

[2]

2. (i) $\log \dfrac{pq}{r} = \log p + \log q - \log r$

[2]

(ii) $\log \dfrac{\sqrt{p}}{r^2} = \log p^{\frac{1}{2}} - \log r^2$

$$= \tfrac{1}{2}\log p - 2\log r$$

[2]

3. (i) $2^x = 7$

$\log 2^x = \log 7$

$x\log 2 = \log 7$

$x = \dfrac{\log 7}{\log 2} = 2.81$ (3 s.f.)

[2]

(ii) $3^{2x} = 5$

$\log 3^{2x} = \log 5$

$2x\log 3 = \log 5$

$x = \dfrac{\log 5}{2\log 3} = 0.732$ (3 s.f.)

[2]

4. (i) $2e^x = 3e^{-x} + 5$

Substituting $y = e^x$: $2y = 3y^{-1} + 5$

Multiplying through by y: $2y^2 = 3 + 5y$

$2y^2 - 5y - 3 = 0$

$(2y+1)(y-3) = 0$

$y = -\tfrac{1}{2}$ or 3

$e^x = -\tfrac{1}{2}$ or 3

Since e^x cannot be negative, $e^x = 3 \quad \Rightarrow x = \ln 3$

[3]

(ii) $\ln(2x+1) = \ln x + 2$

$\ln(2x+1) - \ln x = 2$

$\ln\left(\dfrac{2x+1}{x}\right) = 2$

$\dfrac{2x+1}{x} = e^2$

$2x+1 = xe^2$

$1 = xe^2 - 2x$

$1 = x(e^2 - 2)$

$x = \dfrac{1}{e^2 - 2}$

[3]

5. $500 \times 1.05^n > 1000$

$1.05^n > 2$

$n \log 1.05 > \log 2$

$n > 14.2$

He will be 15 years old when the amount first exceeds £1000.

[4]

6. (i) $N = 20 \times 2^{0.8t}$

When $t = 5$, $N = 20 \times 2^4 = 320$.

[2]

(ii) $20 \times 2^{0.8t} = 2000$

$2^{0.8t} = 100$

$\ln 2^{0.8t} = \ln 100$

$0.8t \ln 2 = \ln 100$

$t = 8.30$

After 8.3 years.

[3]

7. (i) When $t = 0$, $T = 100$, so the initial temperature is 100°C [1]

(ii) $T = 20 + 80e^{-0.5t}$

When $t = 5$, $T = 20 + 80e^{-2.5} = 26.6$

The temperature after 5 minutes is 26.6°.

[2]

(iii) $30 = 20 + 80e^{-0.5t}$

$80e^{-0.5t} = 10$

$e^{-0.5t} = \frac{1}{8}$

$-0.5t = \ln\frac{1}{8}$

$t = 4.16$

After 4.16 minutes.

[3]

(iv) $\frac{dT}{dt} = 80 \times -0.5e^{-0.5t} = -40e^{-0.5t}$

When $t = 0$, $\frac{dT}{dt} = -40e^{0} = -40$

The initial rate of cooling is 40 degrees / minute.

When $t = 2$, $\frac{dT}{dt} = -40e^{-1} = -14.7$ degrees / minute.

The rate of cooling after 2 minutes is 14.7 degrees / minute.

[3]

(v) 20°C [1]

8. (i) $N = ab^t$

$\log N = \log(ab^t)$

$= \log a + \log b^t$

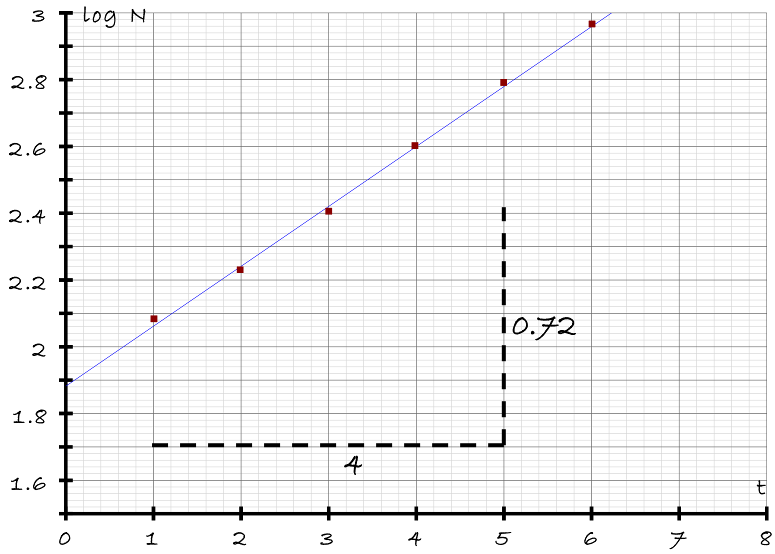
$= \log a + t\log b$

This is the equation of a straight line graph with variables t and log N, so if the relationship is an appropriate model, then plotting log N against t should give an approximate straight line graph.

[2]

(ii)

t	1	2	3	4	5	6
N	120	170	250	400	620	910
log N	2.08	2.23	2.40	2.60	2.79	2.96



[3]

(iii) Equation of graph is $\log N = \log a + t\log b$

Gradient $= \dfrac{0.72}{4} = 0.18$, so $\log b = 0.18 \quad \Rightarrow b = 10^{0.18} \approx 1.5$

Intercept $= 1.88$, so $\log a = 1.88 \quad \Rightarrow a = 10^{1.88} = 76$

[3]

(iv) $N = 76 \times 1.5^t$

After 20 days, $N = 76 \times 1.5^{20} \approx 250000$

If conditions remain the same the estimate is likely to be good, but it could be that the bacteria growth slows if the environment cannot support those numbers.

[2]

7. (i) $y = kx^n$

$\log y = \log(kx^n)$

$= \log k + \log x^n$

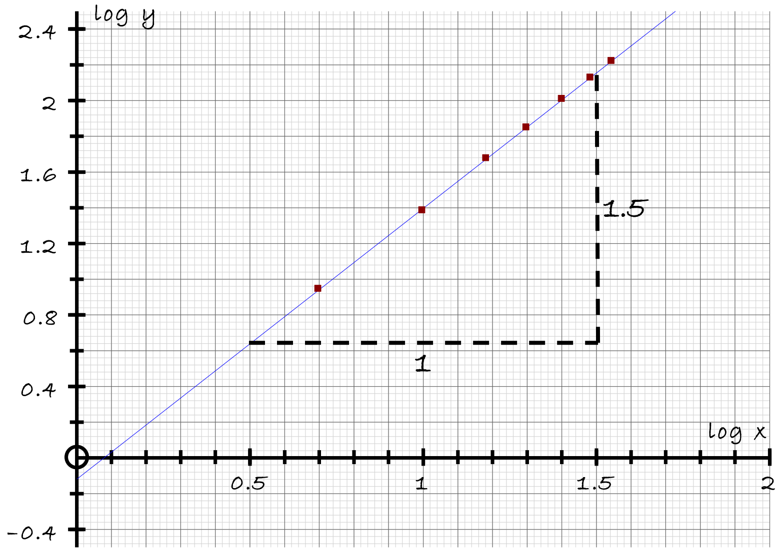
$= \log k + n\log x$

This is the equation of a straight line graph with variables log y and log x, so if the model is an appropriate one then plotting log y against log x will give an approximate straight line graph.

[2]

(ii)

x	5	10	15	20	25	30	35
y	9	24	48	69	102	131	166
log x	0.70	1	1.18	1.30	1.40	1.48	1.54
log y	0.95	1.38	1.68	1.84	2.01	2.12	2.22



[3]

(iii) Equation of graph is $\log y = \log k + n \log x$

Gradient $= \dfrac{1.5}{1}$, so $n \approx 1.5$

Intercept $= -0.12$, so $\log k = -0.12 \quad \Rightarrow k = 10^{-0.12} \approx 0.8$

[3]