Edexcel AS Mathematics Differentiation

Topic Assessment

1. Differentiate with respect to x:

   (i) $y = x^4 + 2x - 3x^3$ (ii) $y = \dfrac{3+2x}{\sqrt{x}}$ (iii) $y = \dfrac{(x-2)(2x+1)}{x^4}$

   [8]

2. Given that $x = (3u+2)(u^2-3)$, find $\dfrac{d^2x}{du^2}$ in terms of u. [3]

3. A curve has equation $y = 2x^3 - 3x^2 - 8x + 9$.

   (i) Find the equation of the tangent to the curve at the point P (2, -3). [3]

   (ii) Find the coordinates of the point Q at which the tangent is parallel to the tangent at P. [3]

4. (i) Find the equation of the normal to the graph $y = x - \dfrac{2}{x^2}$ at the point where $x = 1$. [3]

   (ii) Show that the normal does not meet the curve again. [5]

5. A curve has equation $y = 2x^3 - 6x$.

   (i) Find the points where the curve crosses the x-axis. [2]

   (ii) Find $\dfrac{dy}{dx}$. Hence find the stationary points on the curve. [3]

   (iii) Find $\dfrac{d^2y}{dx^2}$, and use this to determine the nature of the stationary points. [3]

   (iv) Sketch the curve. [2]

6. Find the stationary points of the graph $y = \dfrac{1}{x} + \dfrac{1}{x^2} - \dfrac{1}{x^3}$ and determine the nature of each.

   [7]

7. Two real numbers x and y are such that $2x + y = 100$. Find the maximum value of the product of the two numbers. [5]

8. A cuboid has a square base of length x cm and height y cm.

   (i) Show that the surface area of the cuboid is $2x^2 + 4xy$. [2]

   The surface area of the cuboid is 24 cm^2.

   (ii) Show that $V = 6x - 0.5x^3$ [3]

   (iii) Show that the maximum volume of the box occurs when $x = 2$, and find this maximum volume. [4]

9. Use differentiation from first principles to show that the derivative of x^4 is $4x^3$.

   [4]

Total 60 marks

Edexcel AS Maths Differentiation Assessment solutions

Solutions to Topic Assessment

1. (i) $y = x^4 + 2x - 3x^3 \quad \Rightarrow \frac{dy}{dx} = 4x^3 + 2 - 9x^2$

[2]

(ii) $y = \frac{3+2x}{\sqrt{x}} = 3x^{-\frac{1}{2}} + 2x^{\frac{1}{2}}$

$\frac{dy}{dx} = 3 \times -\frac{1}{2}x^{-\frac{3}{2}} + 2 \times \frac{1}{2}x^{-\frac{1}{2}}$

$= -\frac{3}{2\sqrt{x^3}} + \frac{1}{\sqrt{x}}$

[3]

(iii) $y = \frac{(x-2)(2x+1)}{x^4} = \frac{2x^2 - 3x - 2}{x^4} = 2x^{-2} - 3x^{-3} - 2x^{-4}$

$\frac{dy}{dx} = 2 \times -2x^{-3} - 3 \times -3x^{-4} - 2 \times -4x^{-5}$

$= -4x^{-3} + 9x^{-4} + 8x^{-5}$

$= -\frac{4}{x^3} + \frac{9}{x^4} + \frac{8}{x^5}$

[3]

2. $x = (3u+2)(u^2 - 3) = 3u^3 + 2u^2 - 9u - 6$

$\frac{dx}{du} = 9u^2 + 4u - 9$

$\frac{d^2x}{du^2} = 18u + 4$

[3]

3. (i) $y = 2x^3 - 3x^2 - 8x + 9$

$\frac{dy}{dx} = 6x^2 - 6x - 8$

When $x = 2$, $\frac{dy}{dx} = 6 \times 2^2 - 6 \times 2 - 8 = 24 - 12 - 8 = 4$

Tangent has gradient 4 and passes through (2, -3)

Equation of tangent is $y + 3 = 4(x - 2)$

$y + 3 = 4x - 8$

$y = 4x - 11$

[3]

(ii) When gradient = 4, $6x^2 - 6x - 8 = 4$

$$6x^2 - 6x - 12 = 0$$
$$x^2 - x - 2 = 0$$
$$(x-2)(x+1) = 0$$
$$x = 2 \text{ or } x = -1$$

P is the point where x = 2, so Q is the point where $x = -1$.

When $x = -1$, $y = 2(-1)^3 - 3(-1)^2 - 8(-1) + 9 = -2 - 3 + 8 + 9 = 12$

The coordinates of Q are (-1, 12).

[3]

4. (i) $y = x - \frac{2}{x^2} = x - 2x^{-2}$

$$\frac{dy}{dx} = 1 + 4x^{-3} = 1 + \frac{4}{x^3}$$

When x = 1, $\frac{dy}{dx} = 1 + \frac{4}{1^3} = 1 + 4 = 5$

so gradient of normal is $-\frac{1}{5}$

When $x = 1$, $y = -1$

Equation of normal is $y + 1 = -\frac{1}{5}(x-1)$

$$y + 1 = -\frac{1}{5}x + \frac{1}{5}$$
$$y = -\frac{1}{5}x - \frac{4}{5}$$

(ii) If normal meets curve again, $-\frac{1}{5}x - \frac{4}{5} = x - \frac{2}{x^2}$

$$-x - 4 = 5x - \frac{10}{x^2}$$
$$6x - \frac{10}{x^2} + 4 = 0$$
$$6x^3 + 4x^2 - 10 = 0$$
$$3x^3 + 2x^2 - 5 = 0$$

The normal meets the curve at $x = 1$, so $(x-1)$ is a factor

$$(x-1)(3x^2 + 5x + 5) = 0$$

The discriminant of the quadratic is $25 - 4 \times 3 \times 5 = 25 - 60 = -35 < 0$

So the quadratic has no real roots, and therefore the normal does not meet the curve again.

5. (i) $y = 2x^3 - 6x$

When $y = 0$, $2x^3 - 6x = 0$

$x^3 - 3x = 0$

$x(x^2 - 3) = 0$

$x = 0$ or $x^2 = 3 \Rightarrow x = \pm\sqrt{3}$

So the graph cuts the x-axis at the origin and at the points $(\sqrt{3}, 0)$ and $(-\sqrt{3}, 0)$.

[2]

(ii) $\frac{dy}{dx} = 6x^2 - 6$

At stationary points, $6x^2 - 6 = 0$

$x^2 - 1 = 0$

$(x-1)(x+1) = 0$

$x = 1$ or $x = -1$

When x = 1, $y = 2 \times 1^3 - 6 \times 1 = 2 - 6 = -4$

When x = -1, $y = 2 \times -1^3 - 6 \times -1 = -2 + 6 = 4$

so the stationary points are (1, -4) and (-1, 4).

[3]

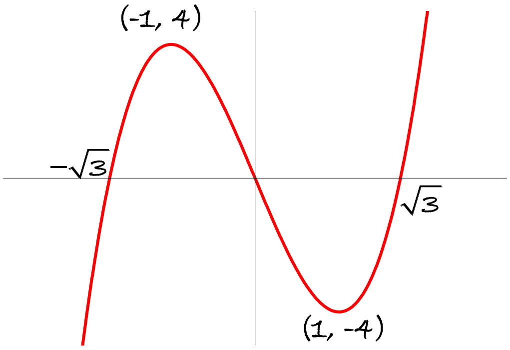
(iii) $\frac{d^2y}{dx^2} = 12x$

When x = 1, $\frac{d^2y}{dx^2} = 12 > 0$ so (1, -4) is a minimum point.

When x = -1, $\frac{d^2y}{dx^2} = -12 < 0$ so (-1, 4) is a maximum point.

[3]

(iv)



[2]

6. $y = \frac{1}{x} + \frac{1}{x^2} - \frac{1}{x^3} = x^{-1} + x^{-2} - x^{-3}$

$\frac{dy}{dx} = -x^{-2} - 2x^{-3} + 3x^{-4} = -\frac{1}{x^2} - \frac{2}{x^3} + \frac{3}{x^4}$

At stationary points, $-\frac{1}{x^2} - \frac{2}{x^3} + \frac{3}{x^4} = 0$

$$\frac{1}{x^4}\left(-x^2 - 2x + 3\right) = 0$$

$$x^2 + 2x - 3 = 0$$

$$(x+3)(x-1) = 0$$

$$x = -3 \text{ or } x = 1$$

When x = -3, $y = -\frac{1}{3} + \frac{1}{9} + \frac{1}{27} = -\frac{5}{27}$

When x = 1, $y = 1 + 1 - 1 = 1$

$\frac{d^2y}{dx^2} = 2x^{-3} + 6x^{-4} - 12x^{-5} = \frac{2}{x^3} + \frac{6}{x^4} - \frac{12}{x^5}$

When x = -3, $\frac{d^2y}{dx^2} = -\frac{2}{27} + \frac{6}{81} + \frac{12}{243} > 0$

When x = 1, $\frac{d^2y}{dx^2} = 2 + 6 - 12 < 0$

so $\left(-3, -\frac{5}{27}\right)$ is a minimum point and (1, 1) is a maximum point.

[7]

7. $2x + y = 100 \quad \Rightarrow y = 100 - 2x$

Product of numbers $P = xy = x(100 - 2x) = 100x - 2x^2$

$\frac{dP}{dx} = 100 - 4x$

At stationary point, $100 - 4x = 0$

$$x = 25$$

P is a quadratic function, and the coefficient of x^2 is negative, so the stationary point must be a maximum point.

When x = 25, y = 50, and P = 1250.

The maximum value of the product of x and y is 1250.

[5]

8. (i) The area of the base and the top are each x^2 square cm, and the other four sides each have area xy square cm.

Total surface area $= 2x^2 + 4xy$.

[2]

(ii) $2x^2 + 4xy = 24$

$x^2 + 2xy = 12$

$2xy = 12 - x^2$

$y = \frac{12 - x^2}{2x}$

$V = x^2 y$

$= x^2 \times \frac{12 - x^2}{2x}$

$= x\left(\frac{12 - x^2}{2}\right) = 6x - 0.5x^3$

[2]

(iii) $\frac{dV}{dx} = 6 - 1.5x^2$

At turning points, $6 - 1.5x^2 = 0$

$\frac{3x^2}{2} = 6$

$x^2 = 4$

$x = \pm 2$

Since x cannot be negative, x must be 2.

$\frac{d^2V}{dx^2} = -3x$

When x = 2, $\frac{d^2V}{dx^2} = -6 < 0$, so x = 2 is a maximum point.

$V = 6 \times 2 - 0.5 \times 2^3 = 12 - 4 = 8$

The maximum volume of the box is 8 cm³.

[4]

9. $f(x) = x^4$

$f(x+h) = (x+h)^4 = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{(x+h) - x}$

$= \lim_{h \to 0} \frac{x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 - x^4}{h}$

$= \lim_{h \to 0} \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h}$

$= \lim_{h \to 0} \left(4x^3 + 6x^2h + 4xh^2 + h^3\right)$

$= 4x^3$