Section 1: Homogeneous differential equations

Section test

1. The general solution of the differential equation

$$\frac{d^2y}{dx^2}+2\frac{dy}{dx}-3y=0$$

is given by:

(a) $y=Ae^{-3x}+Be^{x}$ (b) $y=Ae^{3x}+Be^{-x}$

(c) $y=A\cos 3x+B\sin x$ (d) $y=A\cos x+B\sin 3x$

2. The general solution of the differential equation

$$\frac{d^2y}{dx^2}+16y=0$$

is given by:

(a) $y=(A+Bx)e^{4x}$ (b) $y=A\cos 4x+B\sin 4x$

(c) $y=Ae^{4x}+Be^{-4x}$ (d) $y=A+Be^{-4x}$

3. The general solution of the differential equation

$$\frac{d^2y}{dx^2}+6\frac{dy}{dx}+9y=0$$

is given by:

(a) $y=A\cos 3x+B\sin 3x$ (b) $y=(A+Bx)e^{3x}$

(c) $y=Ae^{3x}+Be^{-3x}$ (d) $y=(A+Bx)e^{-3x}$

4. Find the general solution of the differential equation $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0$.

5. Find the particular solution of the differential equation

$$\frac{d^2y}{dx^2}+6\frac{dy}{dx}+13y=0$$

for which $y = 0$ and $\frac{dy}{dx} = 4$ when $x = 0$.

6. Find the particular solution of the differential equation

$$\frac{d^2x}{dt^2}+2\frac{dx}{dt}+x=0$$

given the conditions $x = 2$ when $t = 0$ and $x = 0$ when $t = 2$.

7. Find the particular solution of the differential equation

$$\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=0$$

given the initial conditions $y = 0$ and $\frac{dy}{dx} = 2$ when $x = 0$.

Edexcel FM Second order DEs 1 section test solutions

8. The movement of a particle is modelled by the differential equation

$$\frac{d^2y}{dt^2} = -4y$$

The period of the motion is

(a) π (b) $\frac{\pi}{2}$

(c) $\frac{1}{\pi}$ (d) $\frac{2}{\pi}$

9. The differential equation

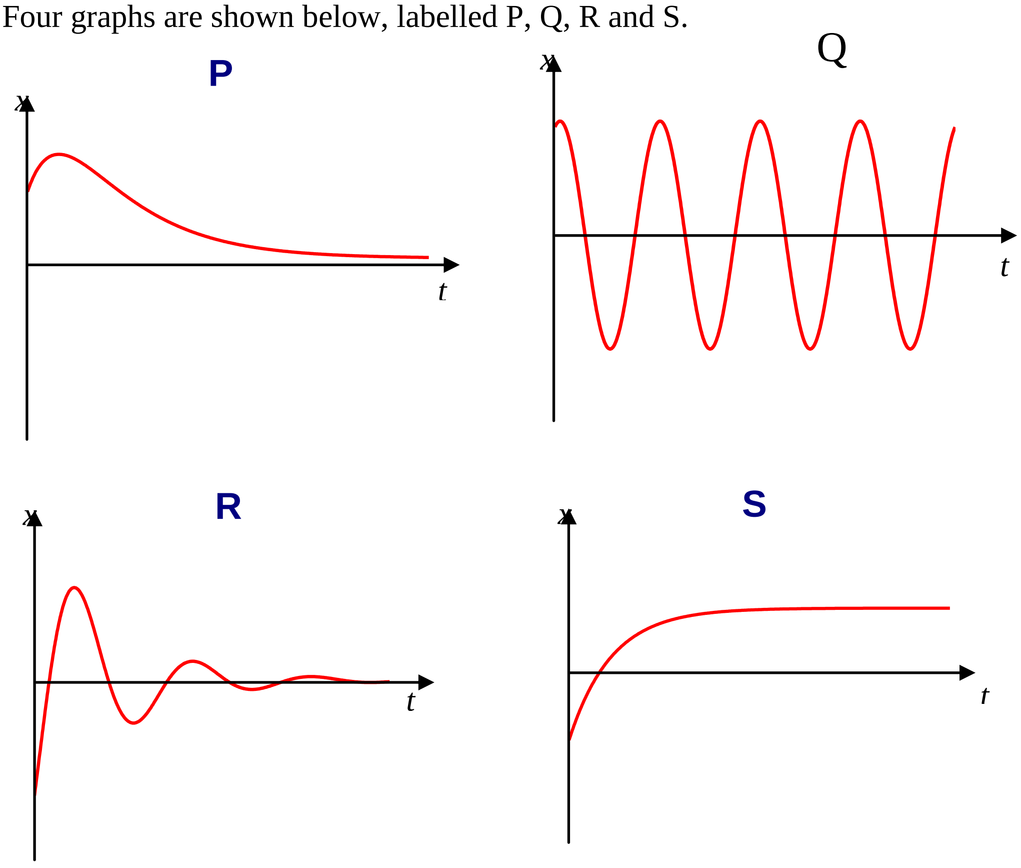
$$\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 4x = 0$$

represents motion which exhibits

(a) critical damping (b) overdamping

(c) underdamping (d) no damping

10. Four graphs are shown below, labelled P, Q, R and S.



Each graph shows a particular solution to one of the following differential equations. Match the differential equation to the graph.

$$4\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 17x = 0 \qquad \frac{d^2x}{dt^2} + 3\frac{dx}{dt} = 0$$

$$\frac{d^2x}{dt^2} + 4x = 0 \qquad \frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2x = 0$$

Edexcel FM Second order DEs 1 section test solutions

Solutions to section test

1. $\frac{d^2y}{dx^2}+2\frac{dy}{dx}-3y=0$

   The auxiliary equation is $m^2+2m-3=0$

   $(m+3)(m-1)=0$

   $m=-3$ or $m=1$

   The general solution is $y=Ae^{-3x}+Be^{x}$

2. $\frac{d^2y}{dx^2}+16y=0$

   The auxiliary equation is $m^2+16=0$

   $m=\pm 4i$

   The general solution is $y=A\cos 4x+B\sin 4x$

3. $\frac{d^2y}{dx^2}+6\frac{dy}{dx}+9y=0$

   The auxiliary equation is $m^2+6m+9=0$

   $(m+3)^2=0$

   $m=-3$

   The general solution is $y=(A+Bx)e^{-3x}$

4. $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0$

   The auxiliary equation is $m^2-2m+2=0$

   $$m=\frac{2\pm\sqrt{4-4\times1\times2}}{2}=\frac{2\pm\sqrt{-4}}{2}=\frac{2\pm 2i}{2}$$

   $m=1\pm i$

   The general solution is $y=e^{x}(A\cos x+B\sin x)$

5. The auxiliary equation is $\lambda^2+6\lambda+13=0$

   $$\lambda=\frac{-6\pm\sqrt{36-4\times1\times13}}{2}=\frac{-6\pm\sqrt{-16}}{2}=\frac{-6\pm 4i}{2}$$

   $\lambda=-3\pm 2i$

   General solution is $y=e^{-3x}(A\sin 2x+B\cos 2x)$

   When $x=0$, $y=0\Rightarrow 0=B$

Edexcel FM Second order DEs 1 section test solutions

$y = Ae^{-3x}\sin 2x$

$\dfrac{dy}{dx} = -3Ae^{-3x}\sin 2x + 2Ae^{-3x}\cos 2x$

When $x = 0$, $\dfrac{dy}{dx} = 4 \Rightarrow 4 = 2A \Rightarrow A = 2$

Particular solution is $y = 2e^{-3x}\sin 2x$

6. The auxiliary equation is $\lambda^2 + 2\lambda + 1 = 0$

$$(\lambda + 1)^2 = 0$$
$$\lambda = -1$$

Repeated root, so general solution is $x = (A + Bt)e^{-t}$

When $t = 0, x = 2 \Rightarrow 2 = A$

When $t = 2, x = 0 \Rightarrow 0 = (2 + 2B)e^{-2} \Rightarrow B = -1$

Particular solution is $x = (2 - t)e^{-t}$

7. The auxiliary equation is $\lambda^2 + 3\lambda + 2 = 0$

$$(\lambda + 1)(\lambda + 2) = 0$$
$$\lambda = -1 \text{ or } -2$$

The general solution is $y = Ae^{-x} + Be^{-2x}$

When $x = 0, y = 0 \Rightarrow 0 = A + B$ (1)

$\dfrac{dy}{dx} = -Ae^{-x} - 2Be^{-2x}$

When $x = 0$, $\dfrac{dy}{dx} = 2 \Rightarrow 2 = -A - 2B$ (2)

(1) + (2): $2 = -B \Rightarrow B = -2, A = 2$

Particular solution is $y = 2e^{-x} - 2e^{-2x}$.

8. The differential equation can be written as $\dfrac{d^2y}{dt^2} + 4y = 0$. Comparing with the SHM equation $\dfrac{d^2x}{dt^2} + \omega^2 x = 0$ gives $\omega = 2$.

The period $= \dfrac{2\pi}{\omega} = \dfrac{2\pi}{2} = \pi$.

9. $\dfrac{d^2x}{dt^2} + 3\dfrac{dx}{dt} + 4x = 0$ models damped harmonic motion, with $\alpha = 3$ and $\omega^2 = 4$.

$\alpha^2 - 4\omega^2 = 9 - 16 < 0$

so the motion exhibits underdamping.

10. $4\frac{d^2x}{dt^2}+4\frac{dx}{dt}+17x=0$

The auxiliary equation is $4\lambda^2+4\lambda+17=0$

$$\lambda=\frac{-4\pm\sqrt{16-4\times4\times17}}{2\times4}=\frac{-4\pm\sqrt{-256}}{8}=\frac{-4\pm16i}{8}$$

$$\lambda=-\tfrac{1}{2}\pm2i$$

General solution is $x=e^{-\frac{1}{2}t}\left(A\sin2t+B\cos2t\right)$

The solution is oscillatory and decaying to zero.

This is graph R.

$\frac{d^2x}{dt^2}+3\frac{dx}{dt}=0$

The auxiliary equation is $\lambda^2+3\lambda=0$

$$\lambda(\lambda+3)=0$$

$$\lambda=0 \text{ or } -3$$

General solution is $x=A+Be^{-3t}$

This solution does not oscillate, and it approaches A as t tends to infinity.

This is graph S (the value of B appears to be negative for this particular solution)

$\frac{d^2x}{dt^2}+4x=0$

The auxiliary equation is $\lambda^2+4=0$

$$\lambda=\pm2i$$

General solution is $y=A\sin2x+B\cos2x$

The solution oscillates, with constant amplitude.

This is graph Q.

$\frac{d^2x}{dt^2}+3\frac{dx}{dt}+2x=0$

The auxiliary equation is $\lambda^2+3\lambda+2=0$

$$(\lambda+2)(\lambda+1)=0$$

$$\lambda=-2 \text{ or } -1$$

General solution is $y=Ae^{-2x}+Be^{-x}$

The solution does not oscillate, and decays to zero as t tends to infinity.

This is graph P.