

## Section 1: Homogeneous differential equations

### Crucial points

- 1. Make sure you write down the auxiliary equation correctly**  
Be especially careful if the differential equation is not written in the usual form: for example if the terms are not in the usual order.
- 2. Remember that if the auxiliary equation has a root,  $\lambda$ , which is zero, this will result in a constant term in the solution.**  
A useful check is that the number of terms in the general solution should be the same as the order of the differential equation.
- 3. Be careful when solving the auxiliary equation, especially where the roots are complex**  
As ever, be careful with your algebra. A mistake at this stage will mess up your whole solution, so check!
- 4. Be sure to use the correct form of the general solution**  
Make sure that you know each of the four cases shown in the table below for second order differential equations.

Roots of auxiliary equation	Form of general solution
Two distinct real roots $\alpha$ and $\beta$	$y = Ae^{\alpha x} + Be^{\beta x}$
One repeated root $m$	$y = (A + Bx)e^{mx}$
Pure imaginary roots $\pm ni$	$y = A \sin nx + B \cos nx$
Complex roots $p \pm qi$	$y = e^{px} (A \sin qx + B \cos qx)$