Edexcel Further Maths Second order DEs

Section 1: Homogeneous differential equations

Exercise level 2

1. A particle is attached to a spring and hangs in equilibrium. It is then pulled a further 0.1m downwards and released from rest. Its displacement x in metres from the equilibrium position after t seconds satisfies the differential equation.

$$\frac{d^2x}{dt^2} + 0.09x = 0$$

(i) Write down the auxiliary equation.
(ii) Find the general solution of the differential equation.
(iii) Find an expression for the velocity v ms^{-1} at time t s.
(iv) Use the initial conditions to find an expression for displacement at time t s.

2. A pendulum swings through small angles. The angle θ that the pendulum makes with the vertical at time t s is given by

$$\frac{d^2\theta}{dt^2} + 6\frac{d\theta}{dt} + 11\theta = 0$$

(i) Find the general solution of the differential equation.
(ii) Initially the pendulum is vertical and the angular velocity $\frac{d\theta}{dt} = 0.5$ s^{-1}. Find the particular solution of the differential equation.
(iii) Find the time, correct to 2 significant figures, at which the pendulum first comes momentarily to rest.
(iv) Describe the behaviour of the pendulum for large values of t.

3. The height of an object above the sea bed is modelled by the differential equation

$$\frac{d^2h}{dx^2} + 5\frac{dh}{dt} + 6h = 0$$

where h is the height in metres at time t s after launch.
The object is launched from the sea bed with an initial velocity of 2 ms^{-1}.
(i) Find an expression for h at time t.
(ii) Show that the model maximum height predicts that the object reaches the maximum height when $t = \ln\left(\frac{3}{2}\right)$ and find the maximum distance.
(iii) Determine whether the model is likely to be a good model for large values of t.

4. The motion of a particle along the x-axis is modelled by the differential equation

$$\frac{d^2x}{dt^2} - 0.4\frac{dx}{dt} + 0.04x = 0$$

where x is displacement from the origin measured in metres and t in seconds.
(i) Find the general solution of the differential equation.
(ii) The initial position of the particle is 3 m in the positive direction and it reaches the origin after 3 s. Find an expression for x.
(iii) Show that the particle is never stationary for positive values of t.
(iv) Explain why the model is not a good model for motion for large values of t.