

Section 1: Homogeneous differential equations

Exercise level 1

1. Find the general solution of each of these differential equations.

(i) $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$

(ii) $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 0$

(iii) $\frac{d^2y}{dx^2} + 16y = 0$

(iv) $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 0$

(v) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

(vi) $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 0$

2. Find the particular solution of each of these differential equations for the given initial conditions.

(i) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 0$ given that $y = 1$ and $\frac{dy}{dx} = 0$ when $x = 0$

(ii) $4\frac{d^2y}{dx^2} + y = 0$ given that $y = 1$ and $\frac{dy}{dx} = 1$ when $x = 0$

(iii) $2\frac{d^2y}{dx^2} - \frac{dy}{dx} - 3y = 0$ given that $y = 0$ and $\frac{dy}{dx} = 1$ when $x = 0$

(iv) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$ given that $y = 1$ and $\frac{dy}{dx} = 2$ when $x = 0$

(v) $4\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + y = 0$ given that $y = 2$ and $\frac{dy}{dx} = 0$ when $x = 0$