Section 1: Polar coordinates and curves

Section test

1. The point P has Cartesian coordinates (-5, 2). Find the polar coordinates of P, giving each one to 3 d.p. where appropriate.

2. The point Q has polar coordinates $\left(3, -\frac{5\pi}{6}\right)$. Find the Cartesian coordinates of Q, giving each one to 3 d.p. where appropriate.

3. Plot the points with polar coordinates $\left(3, \frac{\pi}{6}\right)$, $\left(2, \frac{3\pi}{4}\right)$, $\left(3, -\frac{5\pi}{6}\right)$ and $\left(2, -\frac{\pi}{4}\right)$.
   The shape they form is a

(a) rhombus
(b) rectangle
(c) parallelogram
(d) kite

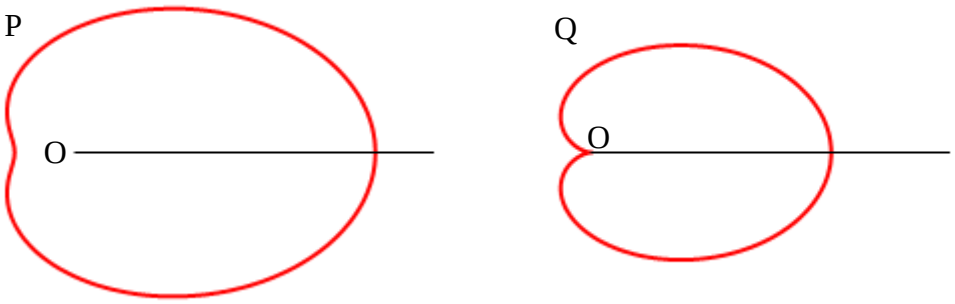
4. A curve has polar equation $2r^2 \sin\theta\cos\theta = 1$.
   The Cartesian equation of the curve is

(a) $2(x^2 + y^2)y = 1$
(b) $xy = 2$
(c) $2xy = 1$
(d) $x^2 + y^2 = y$

5. A curve has Cartesian equation $x^2 + y^2 = 2x$.
   The polar equation of the curve is

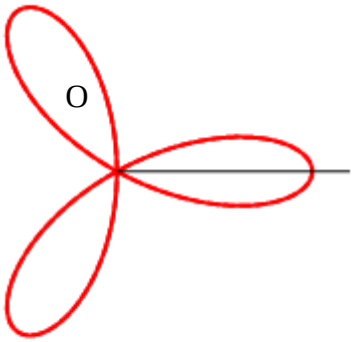
(a) $r = 2\cos\theta$
(b) $r^2 = 2\cos\theta$
(c) $r = 2\sin\theta$
(d) $r^2 = 2\sin\theta$

6. The graphs shown below are not to scale.

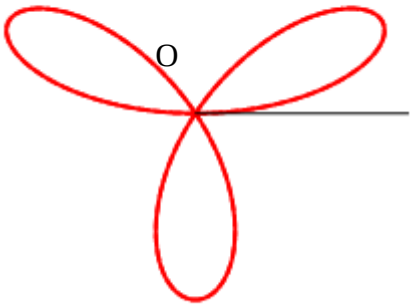


Edexcel FM Polar coordinates 1 section test solutions

R



S



Which diagram shows the graph of the curve $r = 1 + \cos\theta$?
Which diagram shows the graph of the curve $r = \sin 3\theta$?
Which diagram shows the graph of the curve $r = 3 + 2\cos\theta$?
Which diagram shows the graph of the curve $r = \cos 3\theta$?

7. The centre of a circle has polar coordinates $\left(2, \frac{\pi}{4}\right)$. The radius of the circle is 2.

The polar equation of the circle is

(a) $r = 4\sin\theta$
(b) $r = 4\sin(\theta - \frac{\pi}{4})$
(c) $r = 4\cos\theta$
(d) $r = 4\cos(\theta - \frac{\pi}{4})$

Edexcel FM Polar coordinates 1 section test solutions

Section test solutions

1. $r^2 = x^2 + y^2 = 25 + 4 = 29$

   $r = \sqrt{29} = 5.385$

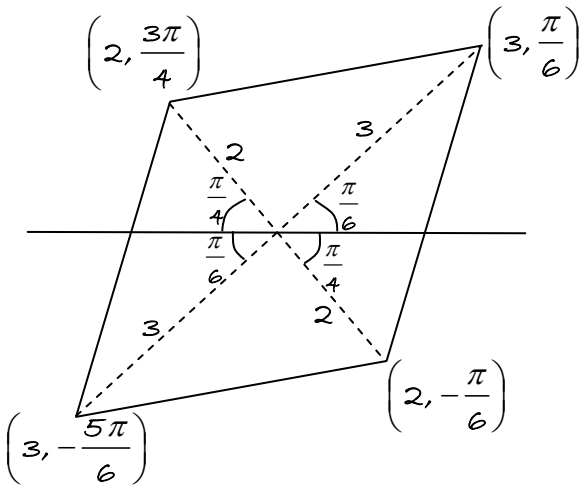
   $\tan\theta = \frac{y}{x} = -\frac{2}{5} \Rightarrow \theta = -0.381$ or 2.761

   Since P is in the second quadrant, the polar coordinates of P are (5.385, 2.761).

2. $x = r\cos\theta = 3\cos\left(-\frac{5\pi}{6}\right) = 3 \times -\frac{\sqrt{3}}{2} = -2.598$

   $y = r\sin\theta = 3\sin\left(-\frac{5\pi}{6}\right) = 3 \times -\frac{1}{2} = -1.5$

   The Cartesian coordinates of Q are (-2.598, - 1.5).

3. 

   By symmetry, the shape has two sets of parallel sides so it is a parallelogram. However, the two sets of sides are not equal in length, so it is not a rhombus, and the diagonals are not equal in length, so it is not a rectangle.

4. $2r^2\sin\theta\cos\theta = 1$

   $2(r\cos\theta)(r\sin\theta) = 1$

   $2xy = 1$

5. $x^2 + y^2 = 2x$

   $r^2 = 2r\cos\theta$

   $r = 2\cos\theta$

Edexcel FM Polar coordinates 1 section test solutions

6. $r = 1 + \cos\theta$

When $\theta = 0$, $r = 2$. The value of r decreases until it reaches zero at $\theta = \pi$.
Since $\cos(-\theta) = \cos\theta$, the graph is symmetrical about the initial line.
This is graph Q.

$r = \sin 3\theta$

The graph is at its maximum value of 1 when $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$ and $-\frac{\pi}{2}$.

This is graph S.

$r = 3 + 2\cos\theta$

When $\theta = 0$, $r = 5$. The value of r decreases until it reaches 1 at $\theta = \pi$.
Since $\cos(-\theta) = \cos\theta$, the graph is symmetrical about the initial line.
This is graph P.

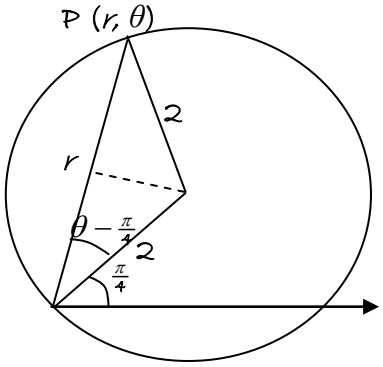
$r = \cos 3\theta$

The graph is at its maximum value of 1 when $\theta = 0, \frac{2\pi}{3}$ and $-\frac{2\pi}{3}$.

This is graph R.

7. From the diagram, $\frac{1}{2}r = 2\cos\left(\theta - \frac{\pi}{4}\right)$

$r = 4\cos\left(\theta - \frac{\pi}{4}\right)$



Alternative method:

Cartesian equation of circle is $\left(x - 2\cos\frac{\pi}{4}\right)^2 + \left(y - 2\sin\frac{\pi}{4}\right)^2 = 4$

$$x^2 - 4x\cos\frac{\pi}{4} + 4\cos^2\frac{\pi}{4} + y^2 - 4y\sin\frac{\pi}{4} + 4\sin^2\frac{\pi}{4} = 4$$

$$x^2 + y^2 - 4x\cos\frac{\pi}{4} - 4y\sin\frac{\pi}{4} = 4 - 4\cos^2\frac{\pi}{4} - 4\sin^2\frac{\pi}{4}$$

$$r^2 - 4r\cos\theta\cos\frac{\pi}{4} - 4r\sin\theta\sin\frac{\pi}{4} = 4 - 2 - 2$$

$$r^2 - 4r\left(\cos\theta\cos\frac{\pi}{4} + \sin\theta\sin\frac{\pi}{4}\right) = 0$$

$$r - 4\cos\left(\theta - \frac{\pi}{4}\right) = 0$$

$$r = 4\cos\left(\theta - \frac{\pi}{4}\right)$$