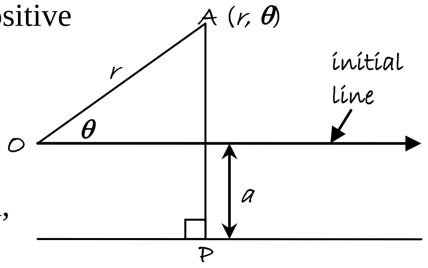
Section 1: Polar coordinates and curves

Exercise level 3

1. The point A moves so that OA = eAP, where e is a positive constant.
   (i) Find the polar equation of the curve in the form $r = \mathrm{f}(\theta)$.
   (ii) Find the Cartesian equation of the curve.
   (iii) Hence show that if e = 1, the curve is a parabola, and give its polar equation.
   (iv) Explore using a graphing program how the curve varies as e varies.



2. Sketch the curve $r = \cos\theta + \tan\theta$ for $0 \le \theta \le \frac{\pi}{2}$.

   Show by considering $\frac{\mathrm{d}r}{\mathrm{d}\theta}$ that r increases as θ increases over this range. Extend this curve in the fourth quadrant between (1, 0) (in polar coordinates) and the origin. When does r = 0?

3. (i) Show that the polar equation $r^2 = 2\operatorname{cosec} 2\theta$ represents the Cartesian equation $y = \frac{1}{x}$.

   (ii) Find the polar equation of the curve $y = \frac{1}{x+y}$ in the form $r^2 = \mathrm{f}(\theta)$.

   (iii) Given $\cos\theta + \sin\theta = \sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right)$, show that this curve can be written as

   $$r^2 = \frac{\operatorname{cosec}\theta\operatorname{cosec}(\theta + \frac{\pi}{4})}{\sqrt{2}}$$

   (iv) Sketch the curve using both its Cartesian and polar equations.