Section 1: Polar coordinates and curves

Exercise level 2

1. (i) A rhombus (a quadrilateral with all sides equal) is centred on the origin. One corner, in polar coordinates (r, θ), is at (2, $\frac{\pi}{3}$). The area of the rhombus is 5. Find the other corners in polar coordinates (r, θ) with $0 \le \theta \le 2\pi$.
   (ii) The corners of a quadrilateral OABC , in polar coordinates (r, θ), are (0, 0), $\left(3, \frac{\pi}{6}\right)$, $\left(4, \frac{\pi}{3}\right)$ and $\left(2, \frac{\pi}{2}\right)$. Find its area.

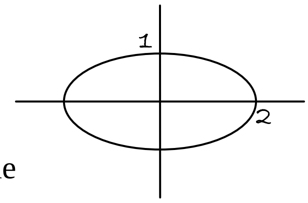
2. (i) Given the curve
$$r = \mathrm{f}(\theta) = \sin\theta + \frac{1}{\sin\theta} \ ,$$
   show $\mathrm{f}(-\theta) = -\mathrm{f}(\theta)$ and $\mathrm{f}(\pi - \theta) = \mathrm{f}(\theta)$.
   (ii) Given also that $y = r\sin\theta$, show $y > 1$.
   (iii)Sketch the curve for $0 < \theta < \pi$. What will the curve be for $\pi < \theta < 2\pi$?
   (iv)Find $\frac{\mathrm{d}r}{\mathrm{d}\theta}$ and hence find the point(s) on the curve closest to O.

3. (i) Show that the curve $r = \frac{1}{\theta} + \frac{1}{\pi - \theta}$ is symmetrical about the line $\theta = \frac{\pi}{2}$.
   (ii) Find $\frac{\mathrm{d}r}{\mathrm{d}\theta}$, and by examining its sign just before and just after $\theta = \frac{\pi}{2}$, show that r has a minimum at $\theta = \frac{\pi}{2}$.
   (iii)Sketch the curve for $0 < \theta < \pi$, given that $y = 1$ is an asymptote to the curve.

4. Sketch the polar curve $r = \cos\theta + k$ for $0 \le \theta \le 2\pi$ for
   (i) $k = 1$ (ii) $k = 3$
   Show that the Cartesian equation of this curve can be written as
$$\sqrt{x^2 + y^2} - \frac{x}{\sqrt{x^2 + y^2}} = k$$
   and, by squaring this, deduce that the curve is approximately $(x-1)^2 + y^2 = k^2$ for large k.

5. A curve has the polar equation $r = a(\cos\theta - \sin\theta)$, where $a > 0$ and $0 \le \theta < 2\pi$.
   (i) Give the values of θ for which $r = 0$.
   (ii) Given that $\cos\theta - \sin\theta = \sqrt{2}\cos\left(\theta + \frac{\pi}{4}\right)$, find the maximum possible value for $|r|$ and the values of θ for which it occurs.
   (iii)Sketch the curve. Show that the curve is a circle.

6. The ellipse shown has Cartesian equation $\frac{x^2}{4}+y^2=1$.



(i) Find the polar equation of the curve in the form $r^2 = \mathrm{f}(\theta)$.

(ii) Give the polar coordinates (r, θ) of the points where the straight line $r=\frac{2}{\sqrt{5}}\sec\theta$ cuts the ellipse.