

Section 3: Further integration

Exercise level 2

1. Find (i) $\int \frac{1}{3+4x+4x^2} dx$ (ii) $\int \frac{1}{\sqrt{3-4x-4x^2}} dx$
(iii) $\int \frac{1}{\sqrt{8+6x-9x^2}} dx$ (iv) $\int \frac{1}{4x^2-4x+5} dx$

2. Evaluate $\int_0^1 \frac{1}{\sqrt{3+4x-4x^2}} dx$.

3. Find

(i) $\int \frac{3x+2}{1+4x^2} dx$ (ii) $\int \frac{3x+2}{\sqrt{1-4x^2}} dx$ (iii) $\int \frac{3x+2}{(1+4x^2)(1-x)} dx$

4. Use a suitable trigonometric substitution to evaluate

(i) $\int_{-\frac{1}{3}}^{\frac{1}{3}} \frac{1}{(4-9x^2)^{\frac{3}{2}}} dx$ (ii) $\int_0^{\frac{3}{2}} \frac{1}{(9+4x^2)^{\frac{3}{2}}} dx$

5. (i) Show that $\int \sec x dx = \ln |\sec x + \tan x| + c$

(ii) $I_k = \int \frac{1}{(1+a^2x^2)^k} dx$

By using a trigonometric substitution, or otherwise, find I_k for

(a) $k=1$ (b) $k=\frac{1}{2}$ (c) $k=\frac{3}{2}$