

## **Section 2: Inverse trigonometric functions**

#### Notes and Examples

These notes contain subsections on:

- The inverse trigonometric functions (reminder)
- Differentiation of functions involving inverse trigonometric functions
- Integration using inverse trigonometric functions

### The inverse trigonometric functions (reminder)

The inverse trigonometric functions arcsin, arccos and arctan (sometimes written as sin<sup>-1</sup>, cos<sup>-1</sup> and tan<sup>-1</sup>) are introduced in A level Mathematics. Remember that since the functions sin, cos and tan are not one-to-one, they do not have inverses, so the inverse trigonometric functions are the inverses of the trigonometric functions defined for a restricted domain, over which they are one-to-one.

In this section you will learn to differentiate the inverse trig functions and to apply this knowledge to integration.

### Differentiation of functions involving inverse trig functions

This section introduces the three standard results:

$$\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}(\arccos x) = \frac{-1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$$

You can prove these by using implicit differentiation.

$$y = \arcsin x$$
  

$$\sin y = x$$
  

$$\cos y \frac{dy}{dx} = 1$$
  

$$\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1 - \sin^2 y}} = \frac{1}{\sqrt{1 - x^2}}$$

The other two results can be proved in a similar way.

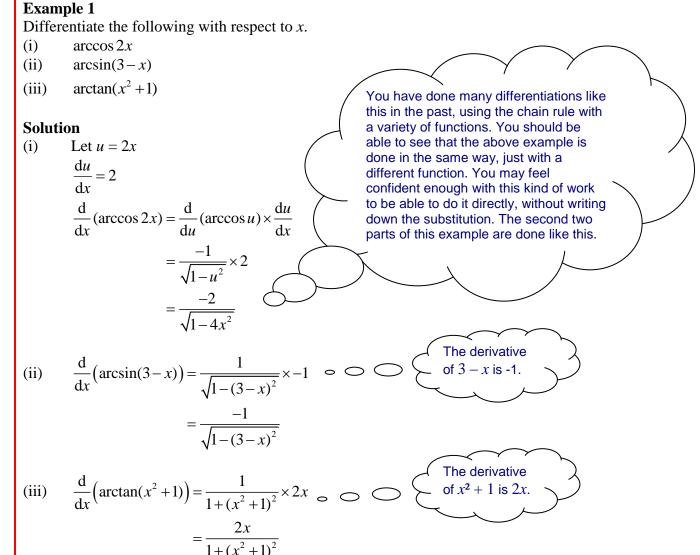
You also need to be able to differentiate other functions involving the inverse trig



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functions, by using the standard results together with the chain rule, as shown in Example 1 below.





#### Integration using inverse trigonometric functions

In this section you learn to apply the two standard integrals

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \arctan \frac{x}{a} + c$$
$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \arcsin \frac{x}{a} + c.$$

Note the following points when using these integrals:

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- If the numerator is a constant other than 1, then no problem: the integral is simply multiplied by this constant.
- You cannot integrate  $\frac{1}{x^2 a^2}$ ,  $\frac{1}{\sqrt{x^2 a^2}}$  or  $\frac{1}{\sqrt{x^2 + a^2}}$  using these standard integrals. However, you will learn to integrate these functions in later work on hyperbolic functions.
- The coefficient of  $x^2$  must be 1 to apply these standard integrals. If it is not, you must take out a factor first, or use a substitution.
- Remember, when taking a factor *b* out of  $\sqrt{a^2 bx^2}$ , you need to take the square root, so that you get  $\sqrt{b}\sqrt{\frac{a^2}{b} x^2}$ .



**Example 2** 

Find  $\int \frac{1}{\sqrt{a^2 - b^2 r^2}} dx$ .

