

Section 1: Improper integrals

Section test

1. Which of the following integrals are improper integrals?

(i) $\int_0^{\infty} x \, dx$ (ii) $\int_0^1 \frac{1}{x} \, dx$
(iii) $\int_0^1 \frac{1}{x+1} \, dx$ (iv) $\int_1^3 \frac{1}{x-2} \, dx$

2. The value of the integral $\int_1^{\infty} \frac{1}{x^{1/3}} \, dx$ is

- (a) $\frac{3}{2}$ (b) 3
(c) $-\frac{3}{2}$ (d) undefined

3. The value of the integral $\int_0^8 \frac{1}{x^{1/3}} \, dx$ is

- (a) $\frac{3}{2}$ (b) 3
(c) 6 (d) undefined

4. The value of the integral $\int_1^{\infty} \frac{1}{x^3} \, dx$ is

- (a) $\frac{1}{2}$ (b) 1
(c) $-\frac{1}{2}$ (d) undefined

5. The value of the integral $\int_0^2 \frac{1}{x^3} \, dx$ is

- (a) $\frac{1}{2}$ (b) $-\frac{1}{8}$
(c) $\frac{1}{8}$ (d) undefined

6. Which of the following integrals can be evaluated?

(i) $\int_{-2}^{\infty} \frac{1}{x^2} \, dx$
(ii) $\int_0^{\infty} \frac{1}{\sqrt{x}} \, dx$

- (a) (i) only (b) (ii) only
(c) both (d) neither

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Solutions to section test

- (i) is an improper integral as one of the limits is infinity
(ii) is an improper integral as the integrand is undefined at $x = 0$
(iii) is not an improper integral (the integrand is undefined at $x = -1$, but this is not between the limits of the integral
(iv) is an improper integral as the integrand is undefined at $x = 2$.

$$\begin{aligned} 2. \int_1^a \frac{1}{x^{1/3}} dx &= \int_1^a x^{-1/3} dx \\ &= \left[\frac{3}{2} x^{2/3} \right]_1^a \\ &= \frac{3}{2} a^{2/3} - \frac{3}{2} \end{aligned}$$

As $a \rightarrow \infty$, $a^{2/3} \rightarrow \infty$, so the integral is undefined.

$$\begin{aligned} 3. \int_a^8 \frac{1}{x^{1/3}} dx &= \int_a^8 x^{-1/3} dx \\ &= \left[\frac{3}{2} x^{2/3} \right]_a^8 \\ &= \frac{3}{2} \times 8^{2/3} - \frac{3}{2} a^{2/3} \\ &= 6 - \frac{3}{2} a^{2/3} \end{aligned}$$

As $a \rightarrow 0$, $a^{2/3} \rightarrow 0$, so the value of the integral is 6.

$$\begin{aligned} 4. \int_1^a \frac{1}{x^3} dx &= \int_1^a x^{-3} dx \\ &= \left[-\frac{1}{2} x^{-2} \right]_1^a \\ &= -\frac{1}{2a^2} + \frac{1}{2} \end{aligned}$$

As $a \rightarrow \infty$, $\frac{1}{2a^2} \rightarrow 0$, so the value of the integral is $\frac{1}{2}$.

$$\begin{aligned} 5. \int_a^2 \frac{1}{x^3} dx &= \int_a^2 x^{-3} dx \\ &= \left[-\frac{1}{2} x^{-2} \right]_a^2 \\ &= -\frac{1}{8} + \frac{1}{2a^2} \end{aligned}$$

As $a \rightarrow 0$, $\frac{1}{2a^2}$ is undefined, so the integral is undefined.

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$$\begin{aligned} 6. \text{ (i)} \int_{-2}^a \frac{1}{x^2} dx &= \int_{-2}^a x^{-2} dx + \int_b^c x^{-2} dx \\ &= [-x^{-1}]_{-2}^a + [-x^{-1}]_b^c \\ &= -\frac{1}{a} - \frac{1}{-2} - \frac{1}{c} + \frac{1}{b} \end{aligned}$$

As $a \rightarrow 0$, $b \rightarrow 0$ and $c \rightarrow \infty$, $\frac{1}{a}$ and $\frac{1}{b} \rightarrow 0$, and $\frac{1}{c}$ is undefined, so the integral is undefined.

$$\begin{aligned} \text{(ii)} \int_0^a \frac{1}{\sqrt{x}} dx &= \int_0^a x^{-1/2} dx \\ &= [2x^{1/2}]_0^a \\ &= 2\sqrt{a} \end{aligned}$$

As $a \rightarrow \infty$, \sqrt{a} is undefined, so the integral is undefined.