Edexcel A Level FM Revision Questions

Hyperbolic functions; Maclaurin series

Question 1

Solve the equation $4\tanh x + \dfrac{13}{\cosh x} = 8$ giving your answer in terms of natural logarithms.

Question 2

(i) Show that $\operatorname{arsinh} x = \ln\left(x + \sqrt{x^2+1}\right)$.

(ii) There are two points on the curve $y = \dfrac{\cosh x}{2 + \sinh x}$ at which the gradient is $\dfrac{1}{9}$.

Show that one of these points is $\left(\ln\left(1+\sqrt{2}\right), \dfrac{\sqrt{2}}{3}\right)$, and find the coordinates of the other point in a similar form.

Question 3

Find the value of

$$\int_1^2 \frac{1}{\sqrt{9x^2-4}}\,\mathrm{d}x$$

giving your answer in an exact logarithmic form.

Question 4

By using the definitions of $\sinh x$ and $\cosh x$, show that

$$\cosh 2x = 1 + 2\sinh^2 x$$

Question 5

(i) Use the substitution $x = 2\sinh u$ to show that

$$\int \sqrt{x^2+4}\,\mathrm{d}x = 2\operatorname{arsinh}\left(\tfrac{1}{2}x\right) + \tfrac{1}{2}x\sqrt{x^2+4} + c$$

where c is an arbitrary constant

(ii) By first expressing $t^2 + 2t + 5$ in a completed square form, show that

$$\int_{-1}^{1} \sqrt{t^2+2t+5}\,\mathrm{d}t = 2\left[\ln\left(1+\sqrt{2}\right) + \sqrt{2}\right]$$

Question 6

By using an appropriate substitution, or otherwise, find

$$\int \sqrt{4x^2 - 25}\,\mathrm{d}x \,.$$

Question 7

(i) Write down the series for $\ln(1+x)$ and the series for $\ln(1-x)$, both as far as the terms in x^5.

(ii) Hence, write down the series for

$$\ln\left(\frac{1+x}{1-x}\right)$$

(iii) Use the series in part (ii) to show that

$$\sum_{0}^{\infty} \frac{1}{(2r+1)4^r} = \ln 3 \,.$$

Question 8

(i) Given that $\mathrm{f}(x) = \arctan\left(\sqrt{2} + x\right)$, find $\mathrm{f}'(x)$ and $\mathrm{f}''(x)$.

(ii) Hence find the Maclaurin series for $\arctan\left(\sqrt{2} + x\right)$ as far as the term in x^2.

(iii) Hence show that, if h is small,

$$\int_{-h}^{h} \arctan\left(\sqrt{2} + x\right)\mathrm{d}x \approx \frac{2}{9}h^3$$

Question 9

Assuming that x^4 and higher powers may be neglected, write down the Maclaurin series approximations for $\sin x$ and $\cos x$, where x is in radians.

Hence obtain an approximation for $\tan x$ in the form $ax + bx^3$.