Edexcel A Level FM Revision Questions

Sequences, series; induction; roots of equations (real)

Question 1

A sequence $\{u_n\}$ is defined by the recurrence relation

$$u_{n+1} = u_n + 2^{n+1}$$

where $u_1 = 5$.

Prove by induction that $u_n = 2^{n+1} + 1$.

Question 2

Prove by induction that

$$1 + 8 + 27 + \ldots + n^3 = \tfrac{1}{4}n^2(n+1)^2$$

Question 3

Show that

$$5^n + 2 \times 11^n$$

is divisible by 3 for all values of $n \geq 0$, $n \in \mathbb{N}$.

Question 4

(i) The equation $x^2 + px + qx + 5 = 0$ has roots α, β, and γ, where

$$\alpha + \beta + \gamma = 3$$
$$\alpha^2 + \beta^2 + \gamma^2 = 4$$

Find p and q.

(ii) The roots of the cubic equation $2x^3 - 5x^2 + ux + v = 0$ are w, $10w$ and $-6w$.

Find the values of the roots and the values of u and v.

Question 5

The roots of the cubic equation $x^3 - 4x^2 + 8x + 7 = 0$ are α, β, and γ.

Find the cubic equation whose roots are $2\alpha + 1$, $2\beta + 1$ and $2\gamma + 1$.

Question 6

Use standard formulae to show that

$$\sum_{r=1}^{n} r^2(3-4r) = \tfrac{1}{2}n(n+1)(1-2n^2)$$

Question 7

(i) Express

$$\frac{1}{r(r+2)}$$

in partial fractions.

(ii) Hence, prove by the method of differences that

$$\sum_{r=1}^{n} \frac{1}{r(r+2)} = \frac{n(An+B)}{4(n+1)(n+2)}$$

where A and B are constants that need to be found.

Question 8

Find the values of A, B and C in the partial fractions and show that $A + B + C = 0$.

$$\frac{2r+5}{(2r-1)(2r+1)(2r+3)} = \frac{A}{2r-1} + \frac{B}{2r+1} + \frac{C}{2r+3}$$

Hence, use the method of differences to find the values of P, Q and R in the expression

$$\sum_{r=1}^{n} \frac{2r+5}{(2r-1)(2r+1)(2r+3)} = P + \frac{Q}{2n+1} + \frac{R}{2n+3}$$