Advanced Mathematics Support Programme®

Edexcel A Level FM Revision Questions

Differential equations; SHM

Question 1

The equation of a curve in the x-y plane satisfies the differential equation

$$(x+1)\frac{dy}{dx} - xy = e^{2x}$$

for $x > -1$.

(i) Show that an integrating factor for this equation is

$$e^{-x}(1+x)$$

and hence find the general solution for y in terms of x.

The curve passes through (0, -3).

(ii) Find the equation of the curve.

Question 2

A raindrop falls from rest through a cloud. Its velocity, v ms^{-1} vertically downwards, at time t seconds after it starts to fall is modelled by the differential equation

$$(t+1)\frac{dv}{dt} + 2v = (t+1)g - 2$$

where g is the acceleration due to gravity and is a constant.

(i) Solve the differential equation and show that $v = \frac{1}{3}(t+1)g - 1 + (1 - \frac{1}{3}g)(t+1)^{-2}$.

In an improved model, the term -2 is replaced by $-2v$, giving the differential equation

$$(t+1)\frac{dv}{dt} + 2v = (t+1)g - 2v$$

(ii) Find the solution to the differential equation subject to the same initial conditions as before.

Question 3

The vertical oscillations of the (undamped) springs of the front suspension of a car can be modelled by the differential equation

$$\frac{d^2y}{dt^2} + 25y = 0$$

where y is the vertical displacement of the top of the suspension at time t.

(i) Find the general solution of this differential equation and describe briefly the behaviour of this system.

The car now travels over a rough surface. The vertical motion can now be modelled by the differential equation

$$\frac{d^2y}{dt^2}+25y=41\cos 4t$$

(ii) By using an appropriate particular integral find the general solution of this differential equation.

Initially $y=1$ and $\frac{dy}{dt}=0$.

(iii) Find the solution subject to these conditions.

(iv) Describe briefly the behaviour of this system.

A refined model for the suspension is given by

$$\frac{d^2y}{dt^2}+8\frac{dy}{dt}+25y=41\cos 4t$$

(v) Again, by finding the complementary and particular integral, find the general solution of this differential equation.

(vi) Describe briefly the behaviour of this system.

Question 4

(i) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2}+6\frac{dy}{dx}+9y=5e^{2x}$$

(ii) Given that when $x=0$, $y=1$ and $\frac{dy}{dx}=0$, find the particular solution.

Question 5

A particle moves in the *x*-*y* plane such that the coordinates (*x*, *y*) metres at time *t* seconds are given by the simultaneous differential equations

$$\frac{dx}{dt}=2x-y+5$$

$$\frac{dy}{dt}=5x-4y+11$$

where $t\geq 0$.

(i) Show that

$$\frac{d^2x}{dt^2}+2\frac{dx}{dx}-3x=9$$

(ii) Find the general solution for x in terms of t.
Hence obtain the corresponding general solution for y.

(iii) Given that $x = 1$, $y = 19$ when $t = 0$, find the particular solutions for x and y in terms of t and sketch graphs of x against t and y against t. Describe the long-term behaviour of the particle.