Edexcel A Level FM Revision Questions

Complex numbers; complex roots of equations

Question 1

If $3+4\mathrm{i}$ is one of the roots of the quartic equation

$$x^4 - 10x^3 + 54x^2 - 130x + 125 = 0$$

find the other three roots.

Question 2

If $\sin 5\theta = A\sin\theta + B\sin^3\theta + C\sin^5\theta$, fnd the values of *A*, *B* and *C*.

Question 3

(i) Show that if $z = \cos\theta + \mathrm{i}\sin\theta$ then $z^n + \dfrac{1}{z^n} = 2\cos n\theta$ and $z^n - \dfrac{1}{z^n} = 2\mathrm{i}\sin n\theta$.

(ii) Show that $\cos^3\theta\sin^4\theta = \frac{1}{64}\left[\cos 7\theta - \cos 5\theta - 3\cos 3\theta + 3\cos\theta\right]$.

Question 4

θ is a real number such that $0 < \theta < \dfrac{\pi}{6}$.

(i) Show that

$$\left(1+\tfrac{1}{2}\mathrm{e}^{3\mathrm{i}\theta}\right)\left(1+\tfrac{1}{2}\mathrm{e}^{-3\mathrm{i}\theta}\right) = \frac{5}{4} + \cos 3\theta$$

(ii) Infinite series C and S are defined by

$$C = \cos 2\theta - \tfrac{1}{2}\cos 5\theta + \tfrac{1}{4}\cos 8\theta - \tfrac{1}{8}\cos 11\theta + \ldots$$

$$S = \sin 2\theta - \tfrac{1}{2}\sin 5\theta + \tfrac{1}{4}\sin 8\theta - \tfrac{1}{8}\sin 11\theta + \ldots$$

By considering $C + \mathrm{i}S$, show that

$$C = \frac{4\cos 2\theta + 2\cos\theta}{5 + 4\cos 3\theta}$$

and find a similar expression for *S*.

Question 5

(i) Find the cube roots of $-2-2\mathrm{i}$ in the form $R\mathrm{e}^{\mathrm{i}\theta}$, where $R>0$ and $-\pi<\theta<\pi$.

These cube roots are represented by the points A, B and C in the Argand diagram, where A is in the first quadrant and ABC are anticlockwise.

M is the midpoint of AB and M represents the complex number *w*.

(ii) Draw on an Argand diagram the points A, B, C and M.

(iii) Find the modulus and argument of *w*.

(iv) Find w^6 in the form $a+b\mathrm{i}$.

Question 6

z is a complex number such that

$$|z-2+3\mathrm{i}|=5$$

(i) Sketch on an Argand diagram the locus of points that satisfy the above equation.

(ii) Find the minimum and maximum value of $|z|$ for points that lie on the locus.

Question 7

z is a complex number that represents points P (*x*, *y*) in the Argand diagram.

Given that

$$|z-1|=2|z+2|$$

show that the locus of points P is given by

$$(x+3)^2+y^2=4.$$

Question 8

z is a complex number such that

$$(z-3)^3=8\mathrm{i}$$

(i) Write each of the solutions of the equation in the form $a+b\mathrm{i}$.

(ii) Sketch these points, labelling them A, B and C, on an Argand diagram such that A has the smallest modulus and B the largest.

(iii) Find the area of the quadrilateral OABCD where O is the origin.