# Section 2: The inverse hyperbolic functions

## Exercise level 3

1. (i) Show that if $y = \operatorname{arcosh} x$, then $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{\sqrt{x^2 - 1}}$.

   (ii) Hence use integration by parts to find $\displaystyle\int \frac{1}{\sqrt{x^2 - 1}} \operatorname{arcosh} x \, \mathrm{d}x$.

   (iii) Using a similar method, or otherwise, find $\displaystyle\int \frac{1}{\sqrt{1 + x^2}} \operatorname{arsinh} x \, \mathrm{d}x$.

2. (i) Show that $\operatorname{arsinh} x > \operatorname{arcosh} x$ for all $x > 1$.
   (ii) Show that $\operatorname{arsinh} x - \operatorname{arcosh} x \to 0$ as $x \to \infty$.
   (iii) Find the smallest integer $n$ such that $\operatorname{arsinh} n - \operatorname{arcosh} n < \ln(1.01)$.

3. You are given that $y = a \operatorname{arcosh} x + b \operatorname{arsinh} x$ and that $y'(2) = \sqrt{8} - \sqrt{6}$ and $y'(3) = \frac{1}{2}\left(\sqrt{8} - \sqrt{6}\right)$.
   Find the values of $a$ and $b$ in exact form.
   Check your answers using a graphing program.