Section 2: The inverse hyperbolic functions

Exercise level 2

1. Evaluate the integral $\int \frac{\mathrm{d}x}{\sqrt{4x^2-16x+32}}$.

2. (i) Given that $c \geq 1$ and $\cosh x = c$, show that $x = \pm \ln\left(c + \sqrt{c^2-1}\right)$.

   (ii) Solve the equation $\sinh^2 x + 3\cosh x = 9$, giving the answers in an exact logarithmic form.

3. (i) Prove that $\operatorname{arsinh} x = \ln\left(x + \sqrt{x^2+1}\right)$.

   (ii) Find $\int_0^2 \frac{1}{\sqrt{3x^2+4}}\mathrm{d}x$, giving your answer in logarithmic form.

4. Using the substitution $2x = \sinh u$, find $\int_0^{\sqrt{2}} \sqrt{4x^2+1}\,\mathrm{d}x$.

5. For each of the following integrals, give the method you would use for different sets of integers k. You do not need to find the integrals.

   (i) $\int \frac{1}{\sqrt{k+x^2}}\mathrm{d}x$ (ii) $\int \frac{1}{\sqrt{k-x^2}}\mathrm{d}x$ (iii) $\int \frac{1}{\sqrt{k+2x+x^2}}\mathrm{d}x$

6. (i) Find $\int \frac{x^{m-1}}{\sqrt{x^{2m}-1}}\mathrm{d}x$.

   (ii) Find similar results involving arsinh and artanh.

7. You are given $\operatorname{artanh} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$ for all x

   and $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots$ for $|x| < 1$.

   (i) Write artanh x as $\frac{1}{2}\ln\left(1+\mathrm{f}(x)\right)$ and expand this as far as the term in x^3.

   (ii) Find the Maclaurin expansion of artanh x as far as the term in x^3, and compare this with your result from (i).