Section 1: Introducing the hyperbolic functions

Section test

If you have a calculator with hyperbolic and inverse hyperbolic functions, do not use these functions for this test (except for checking). In an examination you will be expected to show that you have used the definitions of the hyperbolic and inverse hyperbolic functions where appropriate.

1. Find the value of $\cosh 2$.

2. Find the value of $\tanh 0.5$.

3. The value of $\sinh A$ is $\frac{3}{4}$.
   What is the value of $\cosh A$?
   What is the value of $\tanh A$?

4. Find all the real solutions to the equation
   $$4\cosh x + 3\sinh x = 4$$
   $x = 0$ $x = 1$ $x = \ln 7$ $x = -\ln 7$

5. Differentiate $\sinh^3 x$.

(a) $-3\sinh^2 x \cosh x$ (b) $3\sinh^2 x \cosh x$
(c) $3\sinh^2 x$ (d) $\cosh^2 x$

6. Differentiate $\cosh^2 3x \sinh 3x$
(a) $6\cosh 3x \sinh 3x + 3\cosh^3 3x$ (b) $6\cosh 3x \sinh^2 3x + 3\cosh^3 3x$
(c) $3\cosh 3x \sinh^2 3x + 3\cosh^3 3x$ (d) $3\cosh 3x \sinh 3x + 3\cosh^3 3x$

7. Find $\int x\cosh(x^2+1)\mathrm{d}x$
(a) $-\frac{1}{2}\sinh(x^2+1)+c$ (b) $\sinh(x^2+1)+c$
(c) $\frac{1}{2}\sinh(x^2+1)+c$ (d) $-\sinh(x^2+1)+c$

8. Find $\int_0^{\ln 2} \sinh^2 x \cosh x \,\mathrm{d}x$.

9. Find $\int \mathrm{e}^{2x}\sinh x\mathrm{d}x$
(a) $\mathrm{e}^{2x}\cosh x + c$ (b) $\frac{1}{2}\mathrm{e}^{2x}\cosh x + c$
(c) $\frac{1}{6}\mathrm{e}^{3x} + \frac{1}{2}\mathrm{e}^{x} + c$ (d) $\frac{1}{6}\mathrm{e}^{3x} - \frac{1}{2}\mathrm{e}^{x} + c$

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Solutions to section test

1) $\cosh 2 = \dfrac{e^2 + e^{-2}}{2} = 3.76$

2) $\tanh 0.5 = \dfrac{e^1 - 1}{e^1 + 1} = 0.46$

3) $\cosh^2 A = 1 + \sinh^2 A = 1 + \frac{9}{16} = \frac{25}{16}$

$\cosh A = \dfrac{5}{4}$

$\tanh A = \dfrac{\sinh A}{\cosh A} = \dfrac{3/4}{5/4} = \dfrac{3}{5}$

4) $4\cosh x + 3\sinh x = 4$

$\dfrac{4(e^x + e^{-x})}{2} + \dfrac{3(e^x - e^{-x})}{2} = 4$

$7e^x + e^{-x} = 8$

$7e^{2x} + 1 = 8e^x$

$7e^{2x} - 8e^x + 1 = 0$

$(7e^x - 1)(e^x - 1) = 0$

$e^x = \frac{1}{7}$ or 1

$x = -\ln 7$ or 0

5) $\dfrac{d}{dx}\left(\sinh^3 x\right) = 3\sinh^2 x \cosh x$

6) $\dfrac{d}{dx}\left(\cosh^2 3x \sinh 3x\right) = \cosh^2 3x \times 3\cosh 3x + (2\cosh 3x \times 3\sinh 3x)\sinh 3x$

$= 3\cosh^3 3x + 6\cosh 3x \sinh^2 3x$

7) Let $u = x^2 + 1$

$\dfrac{du}{dx} = 2x \quad \Rightarrow dx = \dfrac{1}{2x}du$

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$$\int x\cosh\left(x^2+1\right)dx = \int x\cosh u \times \frac{1}{2x}du$$
$$= \int \tfrac{1}{2}\cosh u\,du$$
$$= \tfrac{1}{2}\sinh u + c$$
$$= \tfrac{1}{2}\sinh\left(x^2+1\right) + c$$

8) $$\int_0^{\ln 2} \sinh^2 x\cosh x\,dx = \left[\tfrac{1}{3}\sinh^3 x\right]_0^{\ln 2}$$
$$= \tfrac{1}{3}\left(\sinh^3(\ln 2) - \sinh^3 0\right)$$
$$= \frac{1}{3}\left(\frac{e^{\ln 2} - e^{-\ln 2}}{2}\right)^3$$
$$= \frac{1}{3}\times\frac{(2-\frac{1}{2})^3}{8} = \frac{9}{64}$$

9) $$\int e^{2x}\sinh x\,dx = \int e^{2x}\times\tfrac{1}{2}\left(e^x - e^{-x}\right)dx$$
$$= \tfrac{1}{2}\int\left(e^{3x} - e^x\right)dx$$
$$= \tfrac{1}{2}\left(\tfrac{1}{3}e^{3x} - e^x\right) + c$$
$$= \tfrac{1}{6}e^{3x} - \tfrac{1}{2}e^x + c$$

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