Section 1: Introducing the hyperbolic functions

Exercise level 2

1. (i)   Starting from $\cosh x = \frac{1}{2}(e^x + e^{-x})$, show that $\cosh 2x = 2\cosh^2 x - 1$.

    (ii) Show that the two stationary points on the curve $y = 7\sinh x - \sinh 2x$ have y-coordinates $3\sqrt{3}$ and $-3\sqrt{3}$.

    (iii) Show that $\int_0^{\ln 3} (7\sinh x - \sinh 2x)\,dx = \frac{26}{9}$.

2. (i)   Find $\int_0^{\ln a} (12\cosh x - 8\sinh x)\,dx$ in terms of a, simplifying your answer.

    (ii) Solve the equation $12\cosh x - 8\sinh x = 9$, giving the answers in logarithmic form.

    (iii) Show that $12\cosh x - 8\sinh x \geq 4\sqrt{5}$.

3. Show that $\sinh\left(\dfrac{A+B}{2}\right)\sinh\left(\dfrac{A-B}{2}\right) = \frac{1}{2}\cosh A - \frac{1}{2}\cosh B$.

    Hence write $\cosh 4 - \cosh 2$ in the form $k\sinh P\sinh Q$, where k, P and Q are to be found.

4. You are given $y = e^x \sinh x$.

    (i)   Find $\dfrac{dy}{dx}$

        (a)   by using the product rule

        (b)   by using the definition of $\sinh x$ and multiplying out.

    (ii) Show that both methods give the same result.