Section 1: Finding and using Maclaurin series

Section test

1. Find the coefficient of x^3 in the Maclaurin expansion of e^{-3x}.

2. Find the coefficient of x^4 in the Maclaurin expansion of $\cos 2x$.

3. Find the third non-zero term in the Maclaurin expansion of $\ln(1-2x)$.

4. The expansion of $\ln(1+3x)$ is valid for

(a) $-\frac{1}{3} < x \leq \frac{1}{3}$ (b) $-1 < x \leq 1$

(c) $-3 < x \leq 3$ (d) all values of x

5. The expansion of $\cos \frac{1}{2}x$ is valid for

(a) $-2 \leq x \leq 2$ (b) $-1 \leq x \leq 1$

(c) all values of x (d) $-\frac{1}{2} \leq x \leq \frac{1}{2}$

6. Find the first three terms of the Maclaurin series for $(1-2x)e^x$.

7. Find the first three terms of the Maclaurin expansion for $\ln\left(\frac{1+2x}{1-x}\right)$.

8. Find the first three terms in the Maclaurin expansion for $\ln(1+e^x)$.

9. The first four terms in the series expansion of e^x are used with $x = 1$ to find an approximation for e. Find the percentage error in the value obtained.

10. The first three terms in the series expansion of $\ln(1+x)$ are used with $x = 0.2$ to find an approximate value for $\ln 1.2$. Find the percentage error in the value obtained.

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Solutions to section test

1. $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$

   The term in x^3 in the expansion of e^{-3x} is $\frac{(-3x)^3}{3!} = -\frac{27x^3}{6} = -\frac{9}{2}x^3$

   The coefficient of x^3 is $-\frac{9}{2}$.

2. $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots$

   The term in x^4 in the Maclaurin expansion of $\cos 2x$ is $\frac{(2x)^4}{4!} = \frac{16x^4}{24} = \frac{2x^4}{3}$

   The coefficient of x^4 is $\frac{2}{3}$.

3. $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots$

   $\ln(1-2x) = (-2x) - \frac{(-2x)^2}{2} + \frac{(-2x)^3}{3} - \ldots$

   The third non-zero term is $\frac{(-2x)^3}{3} = -\frac{8x^3}{3}$

4. The expansion of $\ln(1+x)$ is valid for $-1 < x \leq 1$
   The expansion of $\ln(1+3x)$ is valid for $-1 < 3x \leq 1$, i.e. $-\frac{1}{3} < x \leq \frac{1}{3}$.

5. The expansion of $\cos x$ is valid for all values of x, so the expansion of $\cos \frac{1}{2}x$ is also valid for all values of x.

6. $e^x = 1 + x + \frac{x^2}{2!} + \ldots$

$$(1-2x)e^x = (1-2x)\left(1 + x + \frac{x^2}{2} + \ldots\right)$$
$$= 1 + x + \frac{x^2}{2} - 2x - 2x^2 + \ldots$$
$$= 1 - x - \frac{3x^2}{2} + \ldots$$

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7. $\ln\left(\frac{1+2x}{1-x}\right) = \ln(1+2x) - \ln(1-x)$

$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \ldots$

$\ln(1+2x) = 2x - \frac{(2x)^2}{2} + \frac{(2x)^3}{3} + \ldots$

$= 2x - \frac{4x^2}{2} + \frac{8x^3}{3} + \ldots$

$= 2x - 2x^2 + \frac{8x^3}{3} + \ldots$

$\ln(1-x) = -x - \frac{(-x)^2}{2} + \frac{(-x)^3}{3} + \ldots$

$= -x - \frac{x^2}{2} - \frac{x^3}{3} + \ldots$

$\ln\left(\frac{1+2x}{1-x}\right) = 2x - 2x^2 + \frac{8x^3}{3} + \ldots - \left(-x - \frac{x^2}{2} - \frac{x^3}{3} + \ldots\right)$

$= 3x - \frac{3x^2}{2} + 3x^3 + \ldots$

8. $f(x) = \ln(1+e^x) \Rightarrow f(0) = \ln 2$

$f'(x) = \frac{e^x}{1+e^x} \Rightarrow f'(0) = \frac{1}{2}$

$f''(x) = \frac{e^x(1+e^x) - e^x e^x}{(1+e^x)^2} = \frac{e^x}{(1+e^x)^2} \Rightarrow f''(0) = \frac{1}{4}$

$f(x) = \ln 2 + \frac{1}{2}x + \frac{\frac{1}{4}x^2}{2!} + \ldots$

$= \ln 2 + \frac{1}{2}x + \frac{1}{8}x^2 + \ldots$

9. $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} \ldots$

$e^1 \approx 1 + 1 + \frac{1}{2} + \frac{1}{6} = \frac{8}{3}$

Percentage error $= \frac{e - 8/3}{e} \times 100 = 1.9\%$

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10. $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots$

$$\ln(1.2) \approx 0.2 - \frac{0.2^2}{2} + \frac{0.2^3}{3}$$

$$= 0.2 - 0.02 + \frac{0.008}{3}$$

$$= 0.182666\ldots$$

$$\text{Percentage error} = \frac{0.182666\ldots - \ln 1.2}{\ln 1.2} \times 100 = 0.19\%$$