Section 1: Finding and using Maclaurin series

Exercise level 2

1. Find $\sqrt[3]{\mathrm{e}}$ using the Maclaurin expansion for e^x as far as the term in x^4. How many significant figures are sensible in your answer?

2. Find the Maclaurin expansion as far as the term in x^4 for
$$y = \frac{2\mathrm{e}^x + x}{\mathrm{e}^x} .$$

3. Find the first four terms of the Maclaurin series for $\mathrm{e}^x \cos x$.

4. (i) Write down the Maclaurin expansions for $\cos\theta$ and $\sin\theta$ as far as the term in θ^3.
   (ii) Find the Maclaurin expansion for $\sin 2\theta$
   (a) from the expansion for $\sin\theta$ from (i)
   (b) using the identity $\sin 2\theta = 2\sin\theta\cos\theta$

5. (i) If $y = \frac{1}{(1-3x)^3}$, find y', y'' and y'''.
   (ii) Hence find the Maclaurin expansion of y as far as the term in x^3.
   (iii) By considering the binomial expansion of $(1-3x)^{-3}$, state the range for x for which this series is valid.

6. $\mathrm{f}(x) = \arctan x^3$
   (i) Find $\mathrm{f}'(x)$, and by expanding and then integrating the result, find the series expansion of f(x) as far as the term in x^{21}.
   (ii) Check this using the Maclaurin series
   $$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots .$$
   stating the range of values of x for which the expansion is valid.

7. Let $\mathrm{f}(x) = \arcsin\left(\frac{3}{5} + x\right)$.
   (i) Find $\mathrm{f}'(x)$ and $\mathrm{f}''(x)$.
   (ii) Given that the Maclaurin series for f(x) begins
   $$\arcsin\tfrac{3}{5} + px + qx^2 + \ldots$$
   find p and q.
   (iii) Use this Maclaurin series to calculate an approximate value of $\int_0^{0.1} \mathrm{f}(x)\,\mathrm{d}x$, giving your answer to 4 decimal places.