Section 1: de Moivre's theorem

Section test

1. If $z=-1+\sqrt{3}\mathrm{i}$, what is the value of z^6?

2. What is the value of $\left(\cos\frac{\pi}{6}+\mathrm{i}\sin\frac{\pi}{6}\right)^{12}$?

3. $\frac{1}{(\cos\theta+\mathrm{i}\sin\theta)^6}$ is the same as which of the following?
   (a) $-\cos 6\theta-\mathrm{i}\sin 6\theta$
   (b) $\cos 6\theta-\mathrm{i}\sin 6\theta$
   (c) $\cos 6\theta+\mathrm{i}\sin 6\theta$
   (d) $-\cos 6\theta+\mathrm{i}\sin 6\theta$

4. How many roots does the equation $z^5-(1+2\mathrm{i})=0$ have?

5. How many **real** roots does the equation $z^8-1=0$ have?

6. How many **real** roots does the equation $z^7-1=0$ have?

7. Which of the following cannot be the argument of a complex number z such that $z^9=-1+\mathrm{i}$?
   (a) $\frac{\pi}{12}$
   (b) $\frac{29\pi}{36}$
   (c) $\frac{11\pi}{36}$
   (d) $\frac{19\pi}{36}$

8. If w is a complex cube root of 1, what is the value of $1+w^4+w^8$?

9. One of the following is a root of the equation $(z+1)^4=-16$. Which one?
   (a) $\sqrt{2}+\left(\sqrt{2}-1\right)\mathrm{i}$
   (b) $\sqrt{2}+\sqrt{2}\mathrm{i}$
   (c) $\sqrt{2}-1+\sqrt{2}\mathrm{i}$
   (d) $\sqrt{2}-1+\left(\sqrt{2}-1\right)\mathrm{i}$

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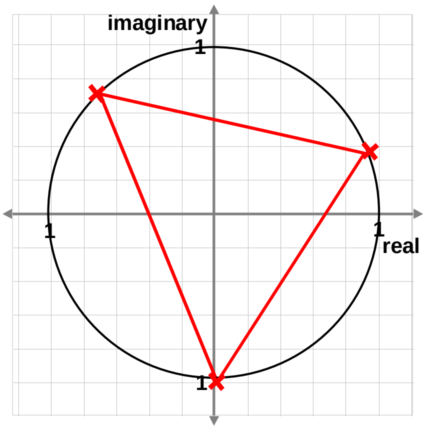
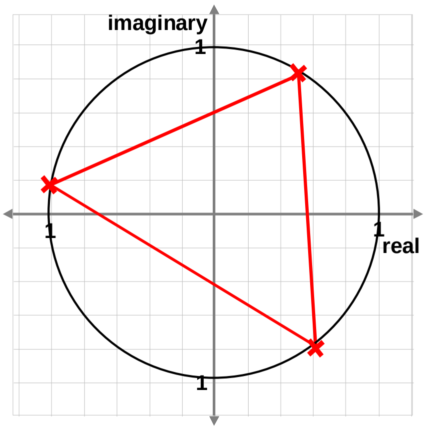
10. Which of the diagrams below represents the set of complex numbers z such that $z^3 = \mathrm{i}$?
(the triangles are intended to be equilateral)

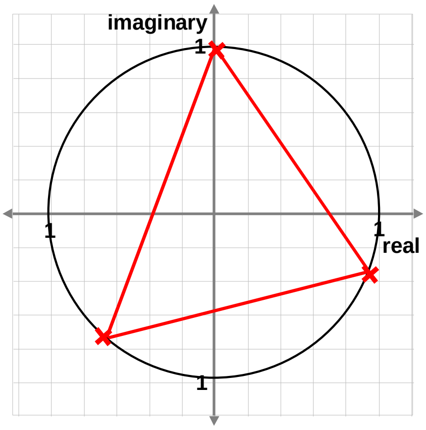
(a)



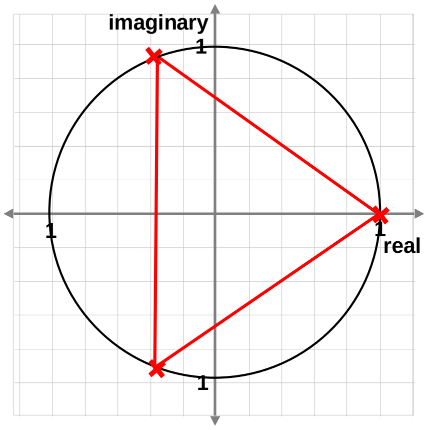
(b)



(c)



(d)



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Solutions to section test

1. $z = -1 + \sqrt{3}i$

   $|z| = \sqrt{1+3} = 2 \quad \Rightarrow |z^6| = 2^6 = 64$

   $\arg z = \tan^{-1}(-\sqrt{3}) = \dfrac{2\pi}{3} \quad \Rightarrow \arg z^6 = 6 \times \dfrac{2\pi}{3} = 4\pi$

   $z^6 = 64(\cos 4\pi + i \sin 4\pi)$

   $\quad = 64$

2. $$\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^{12} = \cos\frac{12\pi}{6} + i\sin\frac{12\pi}{6}$$
   $$= \cos 2\pi + i\sin 2\pi$$
   $$= 1$$

3. $$\frac{1}{(\cos\theta + i\sin\theta)^6} = (\cos\theta + i\sin\theta)^{-6}$$
   $$= \cos(-6\theta) + i\sin(-6\theta)$$
   $$= \cos 6\theta - i\sin 6\theta$$

4. This equation is of degree 5, so it has 5 roots.

5. The roots of this equation form a regular octagon in the Argand diagram. Two of these lie on the real axis, $z = 1$ and $z = -1$.
   So the equation has two real solutions.

6. The roots of this equation form a regular heptagon in the Argand diagram. The only root which lies on the real axis is $z = 1$.
   So the equation has just one real solution.

7. $\arg(-1+i) = \tan^{-1}(-1) = \dfrac{3\pi}{4}$

   The 9th roots of $(-1 + i)$ have arguments given by

   $$\frac{1}{9}\left(\frac{3\pi}{4} + 2r\pi\right) = \frac{(3+8r)\pi}{36} \quad \text{for } r = 0, 1, \ldots 8.$$

   $\dfrac{29\pi}{36}$ does not satisfy this formula, so it cannot be the argument of z.

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8. $1+w^4+w^8 = 1+w^3w+(w^3)^2w^2$

$= 1+w+w^2$

$= 0$

since the sum of the roots of a complex number add up to 0.

9. $(z+1)^4 = -16 = 16(\cos\pi + i\sin\pi)$

$z+1 = 2\left(\cos\frac{(2r+1)\pi}{4} + i\sin\frac{(2r+1)\pi}{4}\right)$

$z = -1+2\cos\frac{(2r+1)\pi}{4} + 2i\sin\frac{(2r+1)\pi}{4}$

When $r = 0$, $z = -1+2\cos\frac{\pi}{4} + 2i\sin\frac{\pi}{4}$

$= -1+\sqrt{2}+\sqrt{2}i$

10. One of the cube roots of i is $-i$, since $(-i)^3 = -1 \times -i = i$.
The cube roots of i therefore form an equilateral triangle with one of the vertices at the point (0, -1).