Edexcel Further Mathematics Complex numbers

Section 2: Applications of de Moivre's theorem

Exercise level 1

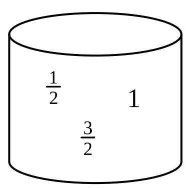
1. Given $(\cos\theta + \mathrm{i}\sin\theta)^n = \cos(n\theta) + \mathrm{i}\sin(n\theta)$, deduce identities for $\cos 2\theta$ and $\sin 2\theta$ in terms of $\cos\theta$ and $\sin\theta$.

2. Express the following in the form $r\mathrm{e}^{\mathrm{i}\theta}$.
   (i) $2 + 2\mathrm{i}$
   (ii) $5 - 5\sqrt{3}\mathrm{i}$

3. If $\mathrm{e}^z = x + \mathrm{i}y$, find x and y in each of the following cases:
   (i) $z = \frac{2\pi\mathrm{i}}{3}$
   (ii) $z = 2 + \frac{\pi\mathrm{i}}{3}$
   (iii) $z = -2 - \frac{\pi\mathrm{i}}{3}$
   (iv) $z = 3 + 2\mathrm{i}$

4. Pick two numbers from the bag (no repeats) and put them into the boxes.

$$z = \square\, \mathrm{e}^{\square\mathrm{i}}$$



   How many different complex numbers can you make?
   Show each of the numbers on an Argand diagram.
   Join up the points to make a convex polygon (i.e. all interior angles are less than 180°). Find the area of the polygon.

5. Write the roots of $z^6 = -64$
   (i) in the form $r\mathrm{e}^{\mathrm{i}\theta}$
   (ii) in the $a + \mathrm{i}b$ form.

6. (i) Find the square roots of 3 + 4i in the form $a + b\mathrm{i}$.
   (ii) Write 3 + 4i in the form $r\mathrm{e}^{\mathrm{i}\theta}$, and find the square roots in this form.
   (iii) Check that your answers agree.