Section 1: de Moivre's theorem

Exercise level 3

1. (i) Find the smallest positive root of $\cos^4 x = \cos 4x$.
   (ii) Show the smallest positive root of $\sin^4 x = \sin 4x$ occurs when $\tan^3 x + 4\tan^2 x - 4 = 0$ has its smallest positive root, given that this root is between 0 and $\frac{\pi}{2}$.

2. (i) Give a complex number z_1 so that z_1 is not real but z_1^7 is real.
   (ii) Give a complex number z_2 so that z_2 is not pure imaginary but z_2^5 is pure imaginary.
   (iii) Show that there is no complex number z_3 so that z_3^7 is real and z_3^5 is pure imaginary.

3. A square has its centre at $a + b$i, and one vertex at $b + a$i. Find the other three vertices. What do the four vertices add up+ to?

4. You are given α = 3 + 4i. Draw the circle in the Argand diagram given by $|z| = |\alpha|$. If $\beta = \alpha^3$, show the cube roots of β on your diagram and label them A, B, C. If P is at the point 2α, find the value of PA × PB × PC
   (i) by using the cosine rule
   (ii) by using $|z|^2 = zz*$.