Section 1: de Moivre's theorem

Exercise level 2

1. (i) Write $\sqrt{3}+\mathrm{i}$ in polar form and hence find $\left(\sqrt{3}+\mathrm{i}\right)^{10}$ in the form a + ib.

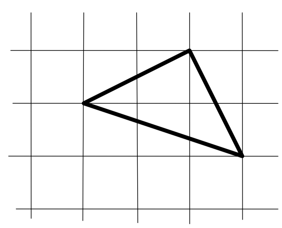
   You are given $z_1=\left(\sqrt{3}-4\mathrm{i}\right)^5$, $z_2=\left(\sqrt{4}-5\mathrm{i}\right)^3$, $z_3=\left(\sqrt{5}-3\mathrm{i}\right)^4$.

   (ii) Which of z_1, z_2 and z_3 has the largest modulus?
   (iii)Which of z_1, z_2 and z_3 has the largest principal argument?

2. In this question, give all answers in an exact form, with arguments in radians between -π and π.
   (i) Find the modulus and argument of 2 – 2i.
   (ii) Hence find the modulus and argument of each of the cube roots of 2 – 2i. Illustrate these cube roots on an Argand diagram.
   The points representing the cube roots are the vertices of a triangle T.
   (iii) Find the modulus and argument of each of the three complex numbers which are represented by the midpoints of the sides of T.
   The three complex numbers in part (iii) are the cube roots of w.
   (iv) Find w, in the form a + bi.

3. (i) You are given that $z=64(\cos\frac{2\pi}{3}+\mathrm{i}\sin\frac{2\pi}{3})$ and that z_2 is the square root of z in the third quadrant, z_3 is the cube root of z in the fourth quadrant and z_6 is the sixth root of z in the second quadrant. Show z_2, z_3 and z_6 on an Argand diagram, and indicate the position of z.
   (ii) Find $z_2z_3z_6$ and comment on your answer.

4. (i) Show from this diagram that
   $\arctan\left(\frac{1}{2}\right)+\arctan\left(\frac{1}{3}\right)=\frac{\pi}{4}$

   (ii) You are given that $z=2+\mathrm{i}$ and $w=\dfrac{3\sqrt{2}}{2}+\dfrac{\sqrt{2}}{2}\mathrm{i}$. Show $|z|=|w|$.
   (iii) Find the exact value of $\arg z+\arg w$.
   (iv) If m is the complex number such that zw is an eighth root of m, find m.

5. (i) Find the six sixth roots of $64(\cos\frac{2\pi}{3}+\mathrm{i}\sin\frac{2\pi}{3})$ and show them on an Argand diagram.
   (ii) Pick three of these roots so that they form the cube roots of a number α. What are the possible values for α?