# Edexcel Further Mathematics Complex numbers

## Section 1: de Moivre's theorem

### Exercise level 1

1. Using de Moivre's theorem, find the value of the following, giving your answers in the form $a + \mathrm{i}b$.

   (i) $(\cos 2\theta + \mathrm{i}\sin 2\theta)^4$

   (ii) $(1+\sqrt{3}\mathrm{i})^{12}$

   (iii) $(1-\mathrm{i})^6$

   (iv) $\left(\cos\frac{\pi}{3} + \mathrm{i}\sin\frac{\pi}{3}\right)^9$

2. $z_1 = \left(\cos\frac{\pi}{4} + \mathrm{i}\sin\frac{\pi}{4}\right)^6$

   $z_2 = \left(\cos\frac{\pi}{6} + \mathrm{i}\sin\frac{\pi}{6}\right)^4$

   (i) Which of $z_1$ and $z_2$ has the larger modulus?
   (ii) Which of $z_1$ and $z_2$ has the larger principal argument?

3. $w_1 = \left(\cos\frac{\pi}{3} + \mathrm{i}\sin\frac{\pi}{3}\right)^{-4}$

   $w_2 = \left(\cos(-\frac{\pi}{4}) + \mathrm{i}\sin(-\frac{\pi}{4})\right)^3$

   (i) Which of $w_1$ and $w_2$ has the larger modulus?
   (ii) Which of $w_1$ and $w_2$ has the larger principal argument?

4. $\cos(\square\theta)\square\mathrm{i}\sin(\square\theta)$

   A plus sign + or a minus sign – is placed into each of the boxes above.
   How many expressions can you create in this way?
   Write them all down, and express each in the form $a(\cos b\theta + \mathrm{i}\sin b\theta)$.
   What do you get if you multiply all these expressions together?
   What do you get if you add them all up?

5. If $\omega$ is a complex cube root of 1, find the value of $(1 + \omega + 2\omega^2)^9$.

6. Write the roots of $z^8 = 1$ in the form:
   (i) $r(\cos\theta + \mathrm{i}\sin\theta)$
   (ii) $a + \mathrm{i}b$

7. If $\omega$ is a complex seventh root of unity, find the other seventh roots of unity in terms of $\omega$.

 