Section 2: More about hypothesis tests

Section test

1. I suspect that a particular coin I have is biased towards heads. In order to investigate this, I toss it 15 times. If X is the number of heads in the 15 tosses, what is the critical region for the hypothesis test conducted at the 5% significance level?

(a) $X \leq 12$
(b) $X \geq 12$
(c) $X \leq 11$
(d) $X \geq 11$

2. I suspect that a particular coin I have is biased. In order to investigate this, I toss it 15 times. If X is the number of heads in the 15 tosses, what is the critical region for the hypothesis test conducted at the 5% significance level?

(a) $X \leq 3$
(b) $X \leq 3$ or $X \geq 12$
(c) $X \geq 12$
(d) $3 < X < 12$

3. A pharmaceutical company claims that its new vaccine is 90% effective. To find out if this claim is too high, a hypothesis test is conducted at the 1% significance level with a sample of 14 patients. Using X to denote the number of patients for whom the vaccine is effective, what is the critical value of X?

4. It is claimed that a coin is fair. In order to test this claim it is tossed 18 times. If X is the number of heads in the 18 tosses, what is the acceptance region for the hypothesis test conducted at the 10% significance level?

5. I suspect that my opponent in a card game may be cheating. To test this, I decided to record the suit of the first card dealt after my opponent had shuffled the pack of cards, and to carry out a hypothesis test to see if the probability that a club was dealt first is different from 0.25. I found that on only one of 20 occasions was the first card dealt a club. At which of the significance levels: 10%, 5%, $2\frac{1}{2}$% and 1%, can I claim that my opponent was cheating?

6. It is claimed that 10% of men can distinguish between butter and margarine, but some people feel that this percentage is too low. Let X be the number of men who can distinguish between butter and margarine. Working at the 5% significance level with a sample of size 12, what is the critical region?

7. A seed manufacturer claims that in a particular variety that he sells there will be one white flower for every three pink flowers. You decide to carry out a hypothesis test to see if this claim is correct, by buying a packet and planting the contents.

   If p is the probability of a white flower, what is the null hypothesis, H_0, which you would use in a hypothesis test?

   If p is the probability of a white flower, what is the alternative hypothesis, H_1, which you would use in a hypothesis test?

   From the packet you bought, you get 10 white and 10 pink flowers.
   Which of the statements below are correct?
   (i) At the 5% significance level, H_0 is rejected.
   (ii) At the 2.5% significance level, H_0 is rejected.

   What is the critical region for this hypothesis test, conducted at the 10% significance level?

Edexcel AS Maths Hyp testing 2 section test solutions

Solutions to section test

1. Let p be the probability of getting a head.

   $H_0 : p = 0.5$
   $H_1 : p > 0.5$

   X is the number of heads in the 15 tosses.
   Need the lowest value of r for which $P(X \geq r) < 0.05$

   $\Rightarrow 1 - P(X \leq r-1) < 0.05$
   $\Rightarrow P(X \leq r-1) > 0.95$

   For $B(15, 0.5)$, $P(X \leq 10) = 0.9408$
   $P(X \leq 11) = 0.9824$

   Lowest value of $r-1$ is 11, so lowest value of r is 12.
   The critical region is $X \geq 12$.

2. Let p be the probability of getting a head.

   $H_0 : p = 0.5$
   $H_1 : p \neq 0.5$

   X is the number of heads in the 15 tosses.
   Since this is a two-tailed test, the critical region has two parts.
   For the lower tail, need the highest value of r for which $P(X \leq r) < 0.025$

   For $B(15, 0.5)$, $P(X \leq 3) = 0.0176$
   $P(X \leq 4) = 0.0592$

   Highest value of r is 3.
   For the upper tail, need the lowest value of r for which $P(X \geq r) < 0.025$

   $\Rightarrow 1 - P(X \leq r-1) < 0.025$
   $\Rightarrow P(X \leq r-1) > 0.975$

   For $B(15, 0.5)$, $P(X \leq 10) = 0.9408$
   $P(X \leq 11) = 0.9824$

   Lowest value of $r-1$ is 11, so lowest value of r is 12.

   The critical region is $X \leq 3$ or $X \geq 12$.

3. Let p be the probability that the vaccine is effective.

   $H_0 : p = 0.9$
   $H_1 : p < 0.9$

   X is the number of patients for whom the vaccine is effective.
   Need the highest value of r for which $P(X \leq r) < 0.01$

   For $B(14, 0.9)$, $P(X \leq 9) = 0.092$
   $P(X \leq 10) = 0.0441$

4. Highest value of r is 9.
   The critical value is 9.

Note that the question asks for the critical value, which is the highest value in the critical region.

Edexcel AS Maths Hyp testing 2 section test solutions

Let p be the probability of getting a head.
$H_0 : p = 0.5$
$H_1 : p \neq 0.5$

X is the number of heads in the 18 tosses.
Since this is a two-tailed test, the critical region has two parts.
For the lower tail, need the highest value of r for which $P(X \leq r) < 0.05$
For $B(18, 0.5)$, $P(X \leq 5) = 0.0481$
$P(X \leq 6) = 0.1189$
Highest value of r is 5.
For the upper tail, need the lowest value of r for which $P(X \geq r) < 0.05$
$\Rightarrow 1 - P(X \leq r - 1) < 0.05$
$\Rightarrow P(X \leq r - 1) > 0.95$
For $B(18, 0.5)$, $P(X \leq 11) = 0.8811$
$P(X \leq 12) = 0.9519$
Lowest value of $r - 1$ is 12, so lowest value of r is 13.

The acceptance region is $6 \leq X \leq 12$.

5. Let p be the probability of dealing a club

$H_0 : p = 0.25$
$H_1 : p \neq 0.25$

X is the number of clubs in the 20 occasions.
For $B(20, 0.25)$, $P(X \leq 1) = 0.0243$
At 10% significance level, reject H_0 since $P(X \leq 1) < 0.05$
At 5% significance level, reject H_0 since $P(X \leq 1) < 0.025$
At $2\frac{1}{2}$% significance level, accept H_0 since $P(X \leq 1) > 0.0125$
At 1% significance level, accept H_0 since $P(X \leq 1) > 0.001$

So H_0 is rejected (i.e. there is evidence to suggest that opponent is cheating) at 5% and 10% levels only.

6. X is the number of men who can distinguish between butter and margarine.
$X \sim B(12, p)$, where p is the probability that a man can distinguish between butter and margarine.

$H_0 : p = 0.1$
$H_1 : p > 0.1$

Need the lowest value of r for which $P(X \geq r) < 0.05$
$\Rightarrow 1 - P(X \leq r - 1) < 0.05$
$\Rightarrow P(X \leq r - 1) > 0.95$

For $B(12, 0.1)$, $P(X \leq 2) = 0.8891$
$P(X \leq 3) = 0.9744$
Lowest value of $r - 1$ is 3, so lowest value of r is 4.
The critical region is $X \geq 4$.

7. The null hypothesis is always of the form "$p = \ldots$"
H_0: $p = \frac{1}{4}$

There is no indication of suspicion that the proportion differs in a particular direction, so this is a two-tailed test.
H_1: $p \neq \frac{1}{4}$

$X = 10$ is in the upper tail.
$P(X \geq 10) = 1 - P(X \leq 9) = 1 - 0.9861 = 0.0139$

This is a two-tailed test, so at the 5% significance level compare this probability with 2.5%.
$P(X \geq 10) < 0.025$, so reject H_0.

At the 2.5% significance level compare this probability with 1.25%.
$P(X \geq 10) > 0.0125$, so accept H_0.

Since this is a two-tailed test, the critical region has two parts.
For the lower tail, need the highest value of r for which $P(X \leq r) < 0.05$
For $B(20, 0.25)$, $P(X \leq 1) = 0.0243$
$P(X \leq 2) = 0.0913$
Highest value of r is 1.
For the upper tail, need the lowest value of r for which $P(X \geq r) < 0.05$
$\Rightarrow 1 - P(X \leq r - 1) < 0.05$
$\Rightarrow P(X \leq r - 1) > 0.95$
For $B(20, 0.25)$, $P(X \leq 7) = 0.8982$
$P(X \leq 8) = 0.9591$
Lowest value of $r - 1$ is 8, so lowest value of r is 9.
The critical region is $X \leq 1$ or $X \geq 9$