Section 1: Introducing hypothesis testing

Section test

1. A consumer organisation claimed that 20% of a particular component produced by an electronics firm was faulty. The manufacturer denied this and said that the proportion of faulty components was lower than 20%. A quality controller was asked to carry out a hypothesis test. She found one faulty component in a random sample of 20 independent components.

   What is the null hypothesis, H_0, which the quality controller used?
   What is the alternative hypothesis, H_1, which the quality controller used?

   The discrete random variable X is the number of faulty components in the sample of 20. What probability must be calculated to complete the test?

   What is the p-value for this test?

   What conclusion did the quality controller reach? Important comment: The significance level should have been stated BEFORE the hypothesis test was conducted and not after the probability has been calculated, as in this question. This question has deliberately been set in this way in order to test whether you understand how to arrive at a conclusion.

(a) Accept H_0 at the 5% significance level but reject H_0 at the 2.5% level
(b) Reject H_0 at the 5% significance level but accept H_0 at the 2.5% level
(c) Reject H_0 at the 10% significance level but accept H_0 at the 5% level
(d) Accept H_0 at the 10% significance level but reject H_0 at the 5% level

2. It is claimed that 10% of men can distinguish between butter and margarine, but some people feel that this percentage is too low. A hypothesis test is carried out on a random sample of ten men.

   What is the null hypothesis, H_0, which is used?
   What is the alternative hypothesis, H_1 which is used?

   Out of the ten men who were tested, three correctly distinguished between butter and margarine. What is the p-value for this test?

   What is the outcome of the test at the 5% significance level?

   In another sample, N men from a sample of 12 men correctly distinguished between butter and margarine, and H_0 was rejected at the 5% significance level
   What are the possible values of N?

Edexcel AS Maths Hyp testing 1 section test solns

Solutions to section test

1. The null hypothesis is always of the form "$p = \ldots$"
The null hypothesis is given by $H_0 : p = 0.2$

The manufacturer claims that p is less than 0.2.
The alternative hypothesis is given by $H_1: p < 0.2$

$X \sim B(20, 0.2)$
The probability required is $P(X \leq 1)$

$P(X \leq 1) = 0.0692$
So the p-value is 0.0692 (3 s.f.)

$P(X \leq 1) = 0.0692$ which is greater than 5% but less than 10%.
If the significance level were 5%, H_0 would be accepted.
If the significance level were 10%, H_0 would be rejected.

2. The null hypothesis is always of the form "$p = \ldots$"
The null hypothesis is given by $H_0 : p = 0.1$

Since some people think that 10% is too low, the alternative hypothesis is that the true probability is greater than 10%.
The alternative hypothesis is given by $H_1: p > 0.1$

Since the alternative hypothesis looks at a larger probability than 10%, the test looks at the probability of 3 or more positive results.
The probability required is therefore $P(X \geq 3)$

$$\begin{aligned} P(X \geq 3) &= 1 - P(X \leq 2) \\ &= 1 - 0.9298 \\ &= 0.0702 \end{aligned}$$

The p-value is 0.0702

$0.0702 > 0.05$, so H_0 is accepted. There is insufficient evidence to suggest that percentage of 10% is too low.

In the sample of 12, since H_0 was rejected at the 5% significance level:

$$\begin{aligned} P(X \geq N) < 0.05 &\Rightarrow 1 - P(X \leq N-1) < 0.05 \\ &\Rightarrow P(X \leq N-1) > 0.95 \end{aligned}$$

From binomial tables with $n = 12$ and $p = 0.1$, the smallest value of r for which $P(X \leq r) > 0.95$ is $r = 3$.
Therefore $N - 1 \geq 3$

$$N \geq 4$$

