Section 1: Introducing the binomial distribution

Section test

1. Which one of the following conditions is NOT true when the binomial distribution is used to model a situation:

(a) there are two possible outcomes
(b) the sample chosen is a random sample
(c) the probabilities are dependent on the outcomes of the previous trials
(d) both outcomes have fixed probabilities, p and q, and $p + q = 1$

2. An industrial process produces items of which 2% are defective. If a random sample of 40 of these is drawn from a large consignment, what is the probability that the sample contains exactly one item that is defective?

3. Assume that the probability of a baby being a boy is 0.48. A family has five children.

   What is the probability that all the children in the family are of the same sex?
   What is the probability that there are more boys than girls?

4. The discrete random variable X is such that $X \sim \text{B}(7, 0.55)$.

   What is $\text{P}(X = 2)$?
   What is $\text{P}(X \geq 5)$?
   What is $\text{P}(X > 1)$?

5. A bag contains 4 red and 6 blue marbles. A marble is chosen at random, its colour noted and then returned to the bag. This procedure happens eleven times.

   What is the probability that exactly 4 red marbles are drawn?
   What is the most likely number of blue marbles drawn?

6. What is the least number of times you need to toss a fair coin so that the probability of having at least one tail is greater than 0.9999?

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Solutions to section test

1. When the binomial distribution is used, each trial is independent. So the probabilities are not dependent on the outcomes of the previous trials.

2. Let X be the number of defective items.
$X \sim B(40, 0.02)$
$P(X = 1) = {}_{40}C_1 \times 0.02 \times 0.98^{39}$
$= 40 \times 0.02 \times 0.98^{39}$
$= 0.364$ (3 s.f.)

3. $P(\text{all boys}) = (0.48)^5$
$P(\text{all girls}) = (0.52)^5$
$P(\text{all the same sex}) = (0.48)^5 + (0.52)^5 = 0.0635$ (3 s.f.)

Let X be the number of boys in the family
$X \sim B(5, 0.48)$
$P(\text{more boys than girls}) = P(X = 3) + P(X = 4) + P(X = 5)$
$= P(X \geq 3)$
$= 1 - P(X \leq 2)$
$= 1 - 0.537$
$= 0.463$

4. $X \sim B(7, 0.55)$
$P(X = 2) = {}_7C_2 (0.55)^2 (0.45)^5 = \frac{7 \times 6}{1 \times 2}(0.55)^2(0.45)^5 = 0.117$

$P(X \geq 5) = 1 - P(X \leq 4)$
$= 1 - 0.684$
$= 0.316$ (3 s.f.)

$P(X > 1) = 1 - P(X \leq 1)$
$= 1 - 0.0357$
$= 0.964$ (3 s.f.)

5. Let X be the number of red marbles drawn
$X \sim B(11, 0.4)$
$P(X = 4) = {}^{11}C_4 (0.4)^4 (0.6)^7 = \frac{11 \times 10 \times 9 \times 8}{1 \times 2 \times 3 \times 4}(0.4)^4(0.6)^7 = 0.236$ (3 s.f.)

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$P(X=3) = {}^{11}C_3(0.4)^3(0.6)^8 = \dfrac{11\times10\times9}{1\times2\times3}(0.4)^3(0.6)^8 = 0.177$

$P(X=4) = 0.236$ (from above)

$P(X=5) = {}^{11}C_5(0.4)^5(0.6)^6 = \dfrac{11\times10\times9\times8\times7}{1\times2\times3\times4\times5}(0.4)^5(0.6)^6 = 0.221$

The most likely number of red marbles drawn is 4, so the most likely number of blue marbles drawn is 7.

6. Let X be the number of tails obtained

$X \sim B(n, 0.5)$

$P(X \geq 1) > 0.9999 \Rightarrow 1 - P(X=0) > 0.9999$

$\Rightarrow P(X=0) < 0.0001$

$0.5^n < 0.0001$

$0.5^{13} = 0.000122$ and $0.5^{14} = 0.0000610$

so n must be at least 14.

Trial and improvement used – logarithms could also be used.