Edexcel AS Mathematics Force and Newton's laws of motion

Section 1: Force diagrams and equilibrium

Notes and Examples

These notes contain subsections on

- Understanding Newton's laws of motion
- Types of force
- Equilibrium
- Force diagrams
- Representing forces by vectors

Understanding Newton's laws of motion

It will help you to understand and remember Newton's laws of motion if you can relate them to your own experience. Try discussing these examples with other people to help clarify your understanding.

Newton's first law: "Every object continues in a state of rest or uniform motion in a straight line unless acted on by a resultant external force."

1. Think about going quickly round a corner in a car. You feel that you are being thrown across the car, but this is not really what is happening. When you go around a corner, your velocity is changing because your direction of motion is changing. If you were not held in the car, by the friction between your body and the car seat, the car doors and your seat belt, you would continue to travel in a straight line, so the sensation of being thrown across the car is an illusion. What is actually happening is that the car is changing direction and if you are to change direction with it, a force must push you round the corner too. This force is provided by the friction between your body and the car seat, the car doors or your seat belt, which prevent you from continuing in a straight path and force you around the corner. The sensation of being thrown to one side is caused by being forced to change direction.

2. Think about doing an emergency stop in a car. You feel as though you are being thrown against the seatbelt (or through the windscreen if you are not wearing a seatbelt). What is actually happening is that your body will obey Newton's first law and continue to move forward with your original velocity, unless a force acts upon you body to slow it down. This force is provided by the seatbelt (or the windscreen). You are not really being thrown forward, you are being forced back.

3. Look at the necks of Formula 1 racing drivers. You will notice that they appear unusually thick in relation to most people. This is because racing cars turn corners extremely quickly and their drivers need to keep their heads as still as possible so that they can see clearly what is happening. To achieve this, their neck muscles must be very strong to force their

heads around the corners without them being thrown to one side. This is why racing drivers have thick necks!

You have probably heard of whiplash injuries, which can happen when a person's head is thrown backwards. They can occur when a car crashes into the back of a stationary car. People in the stationary car often suffer whiplash injuries, though these can be prevented if the car's seats are provided with head rests. Can you explain this using Newton's first law?

Newton's second law: "Resultant force = mass $\times$ acceleration or $\mathbf{F} = m\mathbf{a}$."

1. It takes a heavy lorry much more distance to stop than a car, even though its brakes can probably provide a greater braking force. This is because $\mathbf{a} = \dfrac{\mathbf{F}}{m}$ (from Newton's second law), so for a given force, the larger m is, the smaller **a** is, so a heavy lorry will slow down more slowly than a light car. Another similar example is oil tankers at sea. They can literally take miles to stop because they are so massive.

2. Try throwing a tennis ball as hard as you can, then try to throw a brick as far as you can. The distance something you throw (a projectile) will travel is dependent upon its initial velocity and the faster it is thrown (at a given angle), the further it will go (you will meet this in detail in chapter 6). With your maximum throwing force you will be able to throw the tennis ball at a greater initial speed than the brick because the brick is much heavier than the tennis ball, so the maximum force from your throwing arm will give it a smaller acceleration and hence a smaller initial speed when you let it go. The tennis ball will therefore go much further.
   (we have ignored air resistance and assumed the same angle of throw for both the brick and the tennis ball – both are reasonable assumptions)

Can you use Newton's second law to explain why rugby players tend to be big and heavy?

Newton's third law: "When one object exerts a force on another there is always a reaction which is equal and opposite in direction to the acting force."

1. If this were not the case, you would fall through the floor. When you are standing on the floor, the force of your weight acts vertically downwards. From Newton's first law, without a balancing force you would accelerate down through the floor. When you are standing stationary on the floor, the reaction force from the floor is exactly balancing your weight, so that the forces acting upon you are in equilibrium.

2. To feel a reaction force directly, try punching the wall! Boxers often break their hands by hitting their opponents.

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There is a video (in three parts) at http://www.open.edu/openlearn/science-maths-technology/science/physics-and-astronomy/describing-motion-along-line/content-section-1.1
which will help you to understand how Newton's ideas revolutionised thinking about force and motion.

Types of force

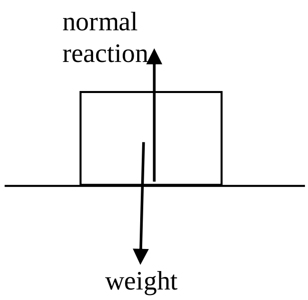
There are a number of types of force you need to consider and whose properties you need to be familiar with.

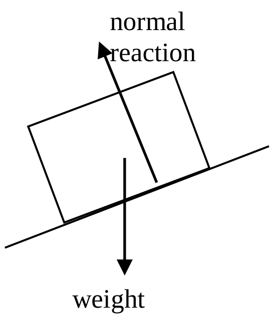
Weight All particles have weight, unless they are defined as being 'light', in which case their weight does not affect the situation significantly and can be ignored. Weight is a force that always acts vertically downwards (towards the centre of the earth).

The weight of an object is not the same as the mass, even though in everyday English the terms are often used interchangeably. Mass is measured in kg; whereas weight is a force and is measured in Newtons. The weight of an object is given by mg, where m is the mass in kg, and g is the acceleration due to gravity.

Resistance This is a force that opposes the motion of a particle. Its direction is always opposite to the direction of motion. It can vary in size in some cases.

Reaction When two objects come into contact with each other, each exerts a force on the other object. The direction of the force is perpendicular to the surface of contact, so the reaction force is sometimes called the normal reaction force.





Friction This is a special type of resistance force. For a stationary object, any frictional force is always at exactly the correct size and direction to keep the object stationary. However, the frictional force has a maximum value. When the resultant force on an object exceeds this maximum, the object will move. The model of friction we use in Mechanics 1 assumes that whilst and object is moving, the frictional force is constant at this maximum value.

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External forces

These are when objects are pushed or pulled. Driving forces and braking forces are examples of external forces.

Tension — Tension is a force that prevents two objects moving away from each other. It is often found in strings and rods. The cross-piece in a pair of step ladders is in tension as it prevents the two sides from separating.
Tension forces 'pull'.

Thrust — Thrust is a force which prevents two objects coming together. It can be found in a rod but not in a string or rope. The legs of a chair provide a thrust force that prevents the seat of the chair falling to the floor. A rod between two objects can provide the thrust force required to keep them apart. A string or rope would just go slack.
Thrust forces 'push'.

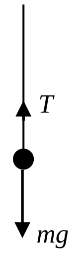
It is important to remember that strings and ropes can exert only tension forces, whereas rods can exert both tensions and thrusts.

Equilibrium

Newton's first law states that "every object continues in a state of rest or uniform motion in a straight line unless acted on by a resultant external force".

An object in a state of rest or moving with constant velocity is said to be in **equilibrium**. This means that if an object is in equilibrium, the resultant force acting on it must be zero.

So, for example, an object hanging from a string in equilibrium has two forces acting on it: its weight acting downwards, and the tension in the string acting downwards.



So the tension, T, must be equal to the weight, mg.

Force diagrams

The diagram above, showing an object hanging from a string, is a very simple example of a force diagram.

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Force diagrams are the fundamental tools for making sense of mechanics questions. Unfortunately many students (usually unsuccessful ones) are reluctant to draw them. Try not to fall into this category! Keep them simple – use plain rectangles – no fancy artwork required! (See the example below)

The idea of a force diagram is to show where forces are said to act, their line of action and, if possible, their direction.

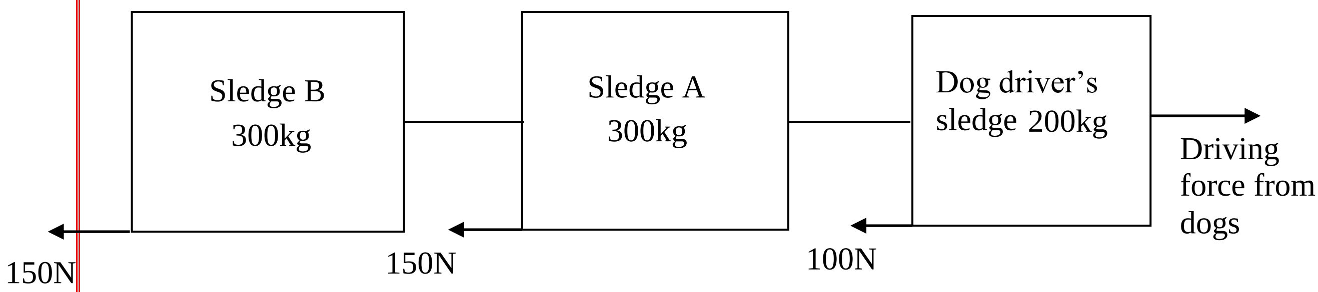
A force diagram should be quite large (10cm) and show all forces acting.

If necessary, do a number of diagrams showing the forces acting on the whole system and on different parts of the system separately.

The example below illustrates this idea. Notice that this example involves motion at constant velocity, which means that there is no resultant force involved.

Example 1

A team of husky dogs is pulling a 'sledge train' over the ice. The 'sledge train' consists of the dog driver's sledge, total mass (including the driver) 200kg and two trailer sledges strung behind it carrying supplies. Each of these trailer sledges has mass 300kg. The resistive forces experienced by each part of the sledge train are shown on the diagram below.



The sledge train is moving at constant velocity.

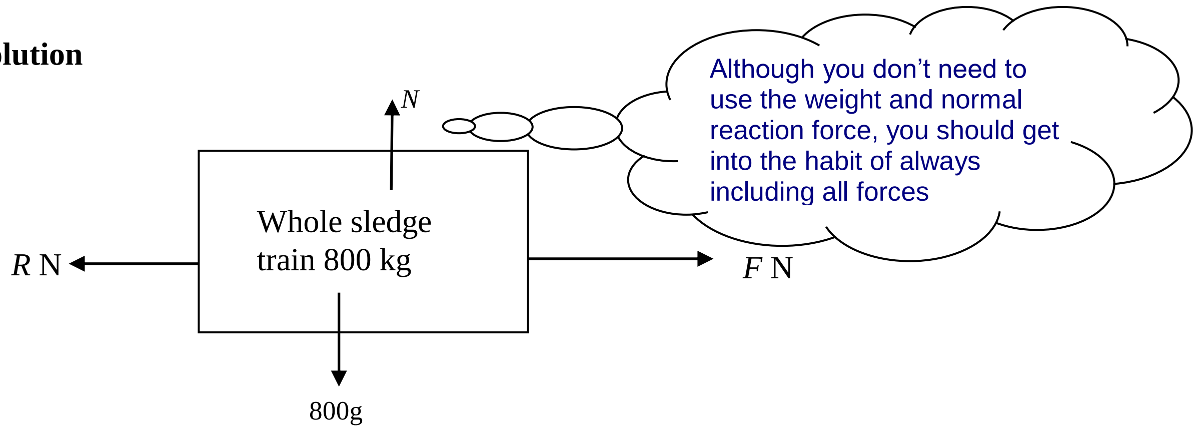
(a) By considering the forces on the system as a whole, calculate the driving force from the dogs.

(b) By considering the forces on the dog driver's sledge, calculate the tension in the coupling between the dog driver's sledge and sledge A.

(c) What is the tension in the coupling between sledge A and sledge B?

Solution

(i)

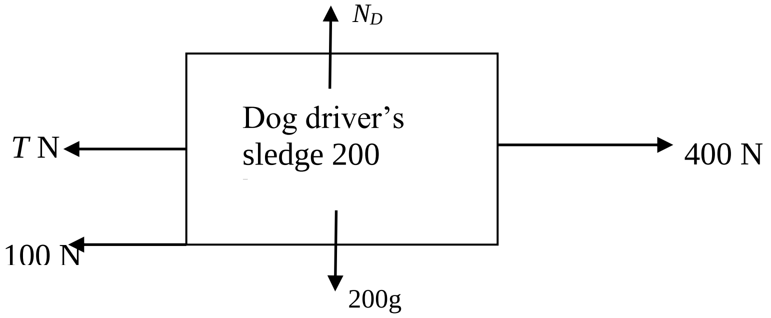


Since the sledge train is moving at constant velocity, the resultant force upon it must be 0, so $R = F$.

$R = 150 + 150 + 100 = 400$
(the combined total resistive force on the sledge train)

So the driving force from the dogs is $R = F = 400$ N.

(ii)

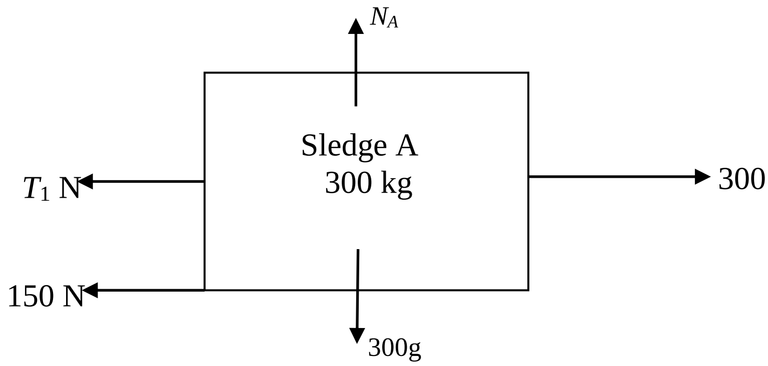


The whole sledge train is moving with constant velocity, so each part of it must be moving with constant velocity, so the forces on the dog driver's sledge must be in equilibrium.

$$T + 100 = 400 \Rightarrow T = 300$$

The tension in the coupling between the dog driver's sledge and sledge A is 300 N.

(iii)



Sledge A must also be in equilibrium, so $T_1 + 150 = 300 \Rightarrow T_1 = 150$.

The tension in the coupling between sledges A and B is 150 N

Make sure you can get the same answer by considering the forces on sledge B

Representing forces by vectors

In two dimensions, forces are represented by vectors. If an object is in equilibrium, the resultant force acting on it is the zero vector, so that it does not move in any direction.

Example 2

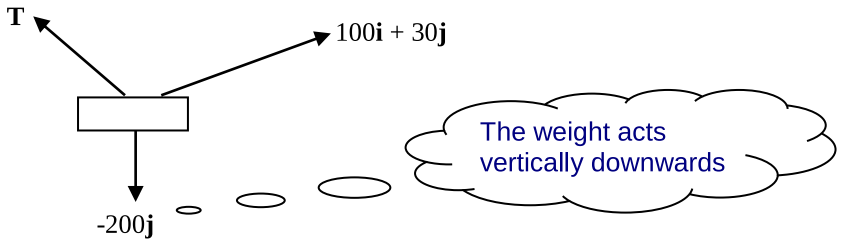
A child is sitting on a swing. The total weight of the child and the swing seat is 200 N. The child's father is holding the swing in equilibrium with a force of $100\mathbf{i} + 30\mathbf{j}$ N.

(i) Find the tension **T** in the chain holding the swing.

(ii) Find the magnitude of this tension and the angle that the chain makes with the vertical.

Solution

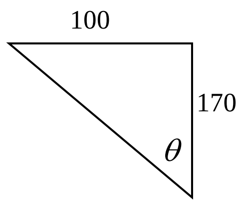
(i)



The swing is in equilibrium, so $\mathbf{T} + 100\mathbf{i} + 30\mathbf{j} - 200\mathbf{j} = \mathbf{0}$

$$\mathbf{T} = -100\mathbf{i} + 170\mathbf{j} \text{ N}$$

(ii) Magnitude $= \sqrt{(-100)^2 + 170^2} = 197$ N (3 s.f.)



$$\tan\theta = \frac{100}{170}$$

$$\theta = 30.5°$$

The chain makes an angle of 30.5° with the vertical.

in the coupling between sledges A and B is 150 N