Section 3: Connected objects

Notes and Examples

These notes contain subsections on
- Solving problems involving connected particles
- Examples

Solving problems involving connected particles

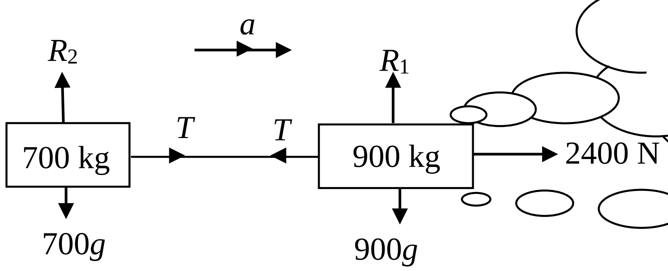
For a problem such as a car towing a caravan, note that there are three possible equations that you can write down: one for each object separately and one for the system as a whole. However, these three equations are not independent: each can be obtained by combining the other two in some way. So you cannot solve a problem with three unknowns, only ones with two unknowns. It is often easier to use the one for the whole system first – this will usually mean that you can avoid using simultaneous equations.

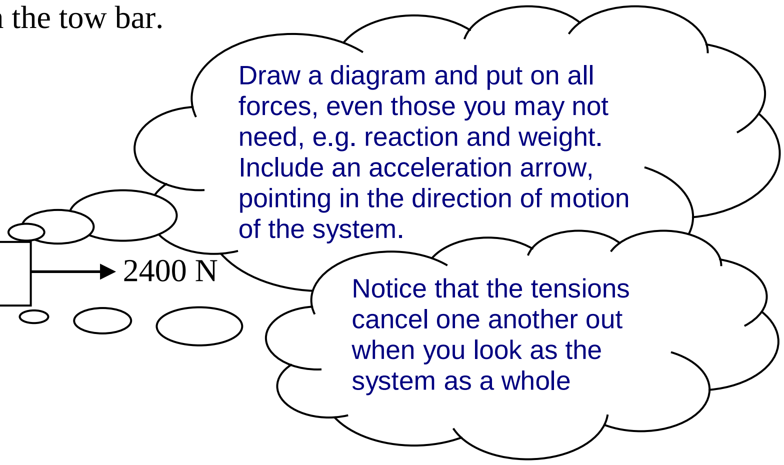
However, when you are dealing with problems involving pulleys, you cannot write down an equation of motion for the whole system, since the use of a pulley means that the particles are moving in different directions. You can only consider the system as a whole when the connected particles are moving in the same line, such as for one vehicle towing another.

Examples

Example 1
A car of mass 900 kg tows a caravan of mass 700 kg along a level road. The engine exerts a forward force of 2400 N and there is no resistance to motion. Find the acceleration produced and the tension in the tow bar.

Solution





For the whole system, in the direction of motion:

$F = ma$

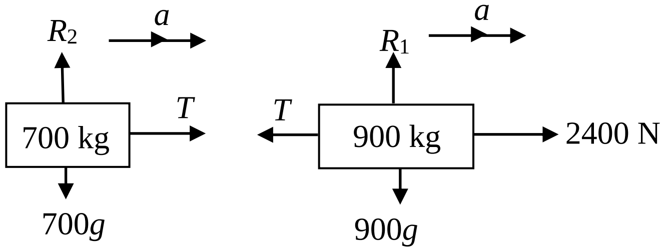
$2400 = (700 + 900)a$

$a = 1.5$

So the acceleration is 1.5 ms^{-2}.

Edexcel AS Maths Force & Newton's laws 3 Notes & Examples

To find the tension in the tow bar, consider the forces on either the car or the caravan. It will work with either. The acceleration of both must be 1.5 ms^{-2} , because they are part of the same connected system.



Consider the forces in the direction of motion:

If you look at the caravan:

$T = 700 \times 1.5$

$T = 1050$

If you look at the car:

$2400 - T = 900 \times 1.5$

$2400 - 1350 = T$

$1050 = T$

So the tension in the tow bar is 1050 N.

Example 2

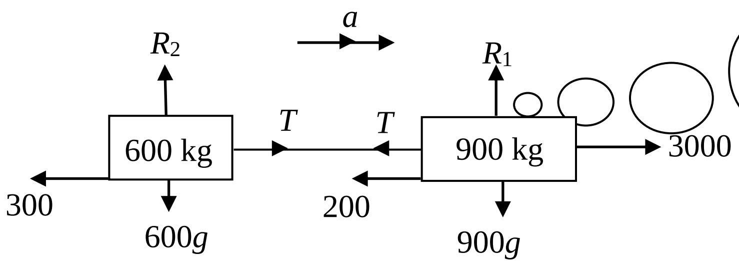
A car of mass 900 kg tows a trailer of mass 600 kg by means of a rigid tow bar. The car experiences a resistance of 200 N and the trailer a resistance of 300 N.

(a) If the car engine exerts a driving force of 3000 N, find the acceleration of the system and the tension in the tow bar.

(b) If the engine is switched off and the brakes are applied to the car, giving a deceleration force of 500 N, what will be the decleration of the car, assuming other resistances are unchanged, and what is the nature and size of the force in the tow bar?

Solution

(a)



Draw a diagram and put on all forces, even those you may not need, e.g. reaction and weight. Include an acceleration arrow, pointing in the direction of motion of the system.

For the whole system: $3000-300-200=(900+600)\times a$

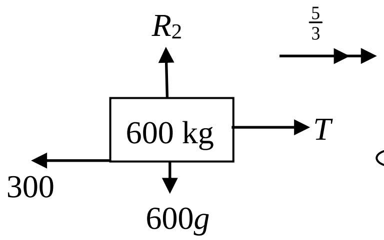
$$2500=1500a$$

$$\frac{2500}{1500}=a$$

$$\tfrac{5}{3}=a$$

The acceleration is $\tfrac{5}{3}$ ms^{-2}.

To find the tension in the tow bar, look at the forces on the trailer. It is slightly easier than looking at the forces on the car, as there is one less force to consider.



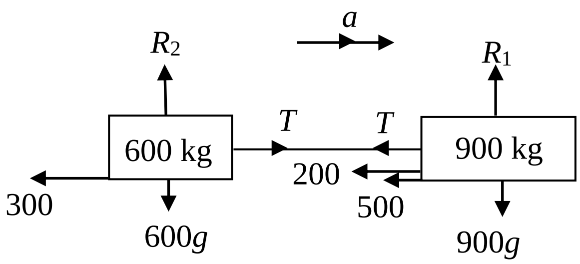
$$T-300=600\times\tfrac{5}{3}$$

$$T=1300$$

So the tension in the coupling is 1300 N.

(b)

Notice that there is now no driving force as the engine has been switched off.



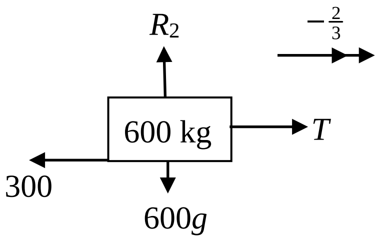
$$0-(500+200+300)=(900+600)\times a$$

$$-1000=1500a$$

$$-\tfrac{2}{3}=a$$

So the acceleration is $-\tfrac{2}{3}$ ms^{-1}.

Again, looking at the forces only acting on the trailer:



$$T-300=600\times-\tfrac{2}{3}$$

$$T=-100$$

So the tension in the tow bar is 100 N and as it is negative it must be keeping the car and trailer apart, so the tow bar is in compression or thrust.

Example 3

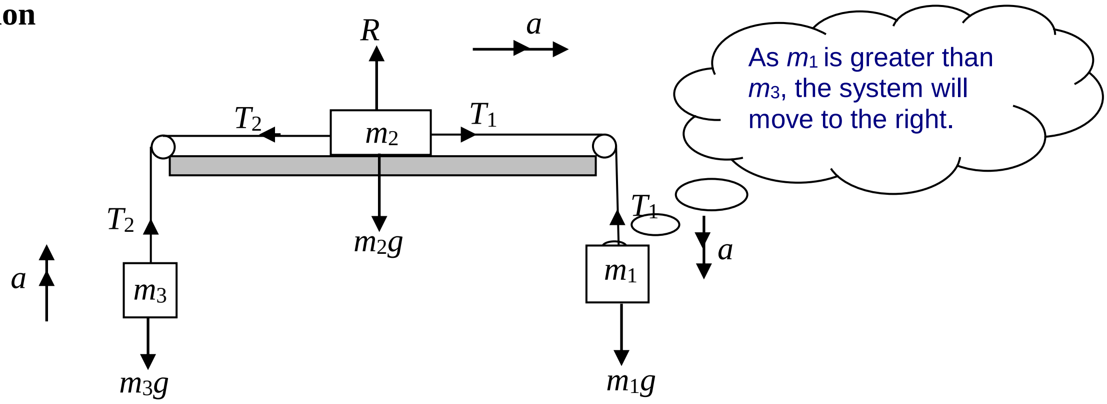
The diagram shows masses of m_1, m_2 and m_3 connected by a light inextensible string. Masses m_1 and m_3 hang vertically with $m_1 > m_3$. Mass m_2 lies on a smooth surface.

(i) Write down equations of motion for each of the masses.

(ii) In the case $m_1 = 8$ kg, $m_2 = 10$ kg and $m_3 = 5$ kg, find the acceleration of the system.

Solution

(i)



Mass m_1 moves down, so the equation of motion for m_1 is

$$m_1 g - T_1 = m_1 a$$

Mass m_3 moves up, so the equation of motion for m_3 is

$$T_2 - m_3 g = m_3 a$$

Mass m_2 moves to the right so T_1 must be greater than T_2 so the equation of motion for m_2 is

$$T_1 - T_2 = m_2 a$$

(ii) $m_1 = 8$, $m_2 = 10$, $m_3 = 5$.

The equations are:

$$8g - T_1 = 8a \quad \Rightarrow T_1 = 8g - 8a$$
$$T_2 - 5g = 5a \quad \Rightarrow T_2 = 5g + 5a$$
$$T_1 - T_2 = 10a$$

Substituting the first two equations into the third:

$$8g - 8a - (5g + 5a) = 10a$$
$$3g = 23a$$
$$a = \frac{3 \times 9.8}{23} = 1.28 \text{ (3 s.f.)}$$

The acceleration of the system is 1.28 ms^{-2} (3 s.f.)