# Section 2: Velocity and acceleration

## Notes and Examples

These notes contain subsections on

- **Acceleration**
- **Distances and displacements from velocity-time graphs**

## Acceleration

Acceleration is a measure of how much velocity is changing. This means it can affect both the speed and direction of motion. In this section you only look at motion along a straight line, so only two directions are possible, either forwards or backwards.

An acceleration of 2 $\text{ms}^{-2}$ means that the velocity of a particle increases by 2 $\text{ms}^{-1}$ every second (by 2 metres per second per second).
For example, if a car has an initial velocity of 6 $\text{ms}^{-1}$ and an acceleration of 2 $\text{ms}^{-2}$, then after 1 second its velocity will be 8 $\text{ms}^{-1}$, after 2 seconds 10 $\text{ms}^{-1}$ and after 3 seconds 12 $\text{ms}^{-1}$ etc.

If a particle has a negative acceleration but a positive velocity, then it will slow down to a stop and then move in the opposite direction, with its speed steadily increasing.

Take care with the word deceleration. It is probably better not to use it. Use negative accelerations instead!

Take care that the units of acceleration are $\text{ms}^{-2}$. This is usually read as 'metres per second squared', or sometimes as 'metres per second per second'.

## Velocity-time graphs

Note the following points about velocity-time graphs.

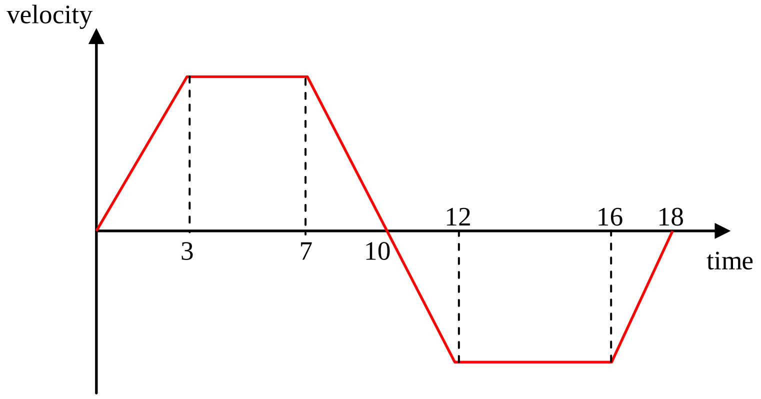
- The gradient of a velocity-time graph is given by
$$\frac{\text{change in velocity}}{\text{time}}$$
which is equal to the acceleration. You have to be careful in interpreting the sign of the gradient: if it is positive, then either the velocity is positive and it is getting more positive, i.e. speeding up, or it is negative and is getting less negative, i.e. slowing down. If the gradient is negative, then either the velocity is positive and is getting less positive, i.e. slowing down, or the velocity is negative and is getting more negative, i.e. speeding up.

- A straight line on a velocity-time graph indicates that the acceleration is constant. If the gradient is positive then the acceleration is positive and if the gradient is negative then the acceleration is negative.

- A horizontal line on a velocity-time graph indicates that the velocity or speed is not changing: i.e. the object is moving at a constant speed.

- Parts of a velocity-time graph below the time axis indicate that the velocity is negative: i.e. the object is moving in the negative direction (although the displacement may still be positive).

The diagram below shows a velocity-time graph.




The straight line from $t = 0$ to $t = 3$ indicates that the particle is accelerating with constant acceleration. The horizontal line from $t = 3$ to $t = 7$ indicates that it is moving with constant speed. The line from $t = 7$ to $t = 10$ indicates that it is moving with constant negative acceleration, i.e. it is slowing down, and at $t = 10$ it is instantaneously stationary.

From $t = 10$ to $t = 12$, the particle has negative velocity and negative acceleration. Although negative acceleration is associated with the idea of slowing down, because the velocity is negative it is in fact becoming more negative, so the velocity is decreasing but its magnitude (the speed) is increasing.

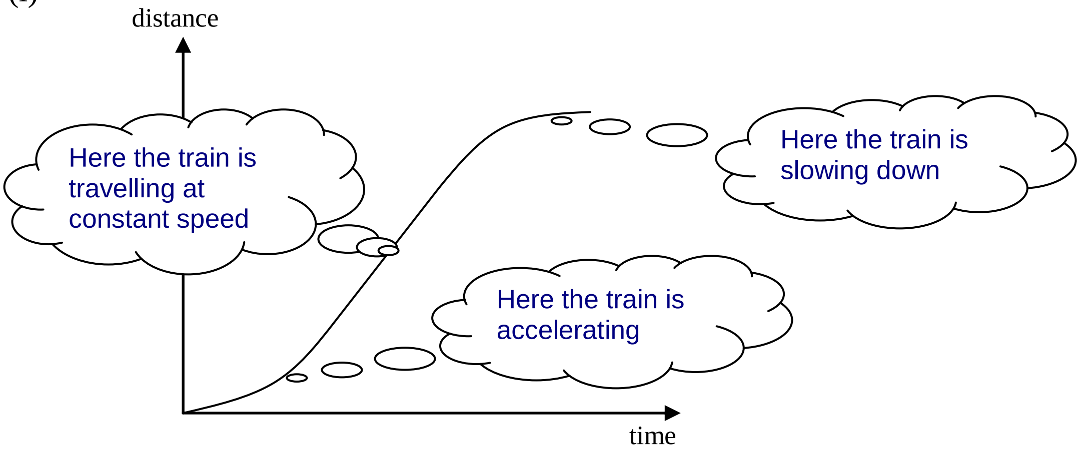
From $t = 12$ to $t = 16$, the velocity is constant. It is negative, meaning that the particle is still moving in a negative direction.

From $t = 16$ to $t = 18$, the particle has negative velocity and positive acceleration. The velocity is becoming less negative, so the velocity is increasing but its magnitude (the speed) is decreasing.

**Example 1**
When a local train leaves a station, it accelerates at a uniform rate of 3 $ms^{-2}$ to its maximum speed of 60 $ms^{-1}$. It then maintains this speed for 2 minutes before slowing down uniformly to a halt at the next station. The whole journey takes 3 minutes.

(i) Sketch a distance-time graph for the journey.
(ii) Find the time the train takes to reach its maximum speed.
(iii) Draw the velocity-time graph for the journey.
(iv) What is the acceleration of the train in the last part of the journey?
(v) Draw the acceleration-time graph for the journey.

**Solution**
(i)



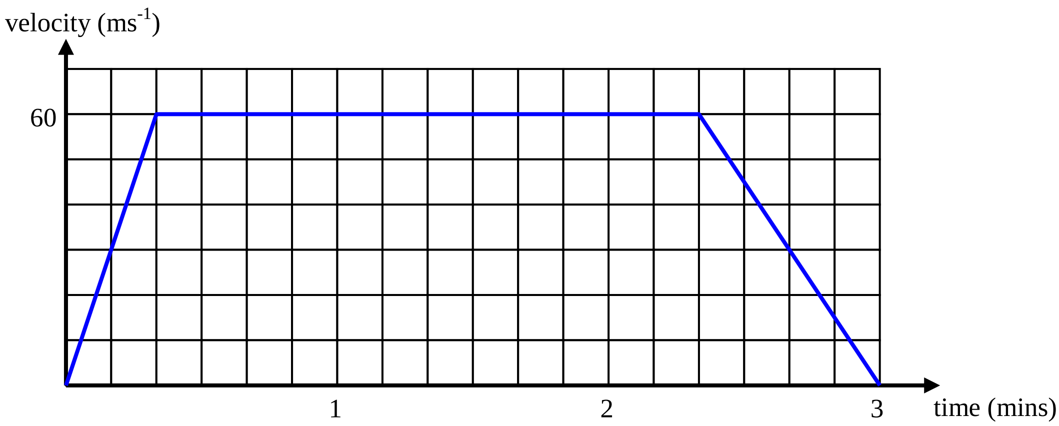

(ii) Acceleration $= \dfrac{\text{change in velocity}}{\text{time}}$

$$3 = \frac{60-0}{t}$$

$$t = \frac{60}{3} = 20$$

The train takes 20 seconds to reach its maximum speed.

(iii)

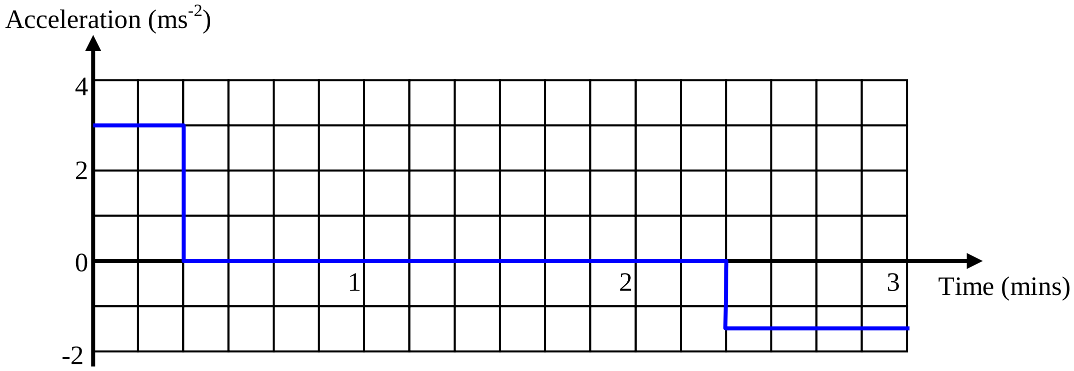



(iv) The last part of the journey takes 40 seconds, and the velocity changes from 60 $ms^{-1}$ to 0.

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{time}}$$
$$= \frac{0-60}{40}$$
$$= -1.5$$

The acceleration is -1.5 ms$^{-2}$.

(v)

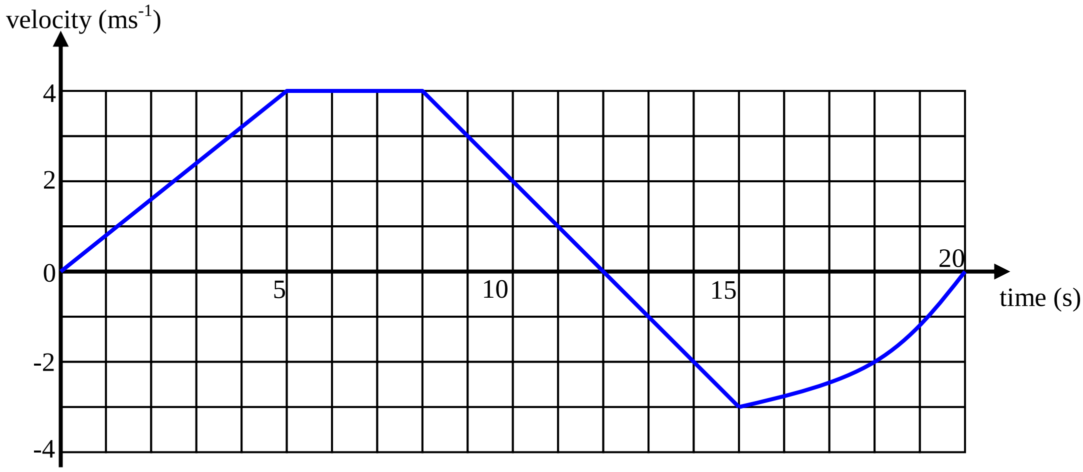



## Distances and displacements from velocity-time graphs

The area under a velocity–time graph is usually equal to the distance a particle travels in the given time period, as long as the line does not cross the time axis (which is the same as saying the velocity is always positive). However, if the graph crosses the time axis (which is the same as saying the velocity becomes negative) as in example 1.2 on page 14, the situation changes so that the distance travelled is equal to the sum of the areas between the graph and the time axis, disregarding the negative sign, whereas the displacement is equal to the sum of the areas, incorporating the negative sign.

If the velocity-time graph is made up of a series of curves, then the area can be approximated by counting squares or by approximating the curve by straight lines to produce trapezia (as for the trapezium rule in Core 2).

**Example 2**
The diagram shows the velocity-time graph for the journey of a particle moving in a straight line.




(i) What is the acceleration of the particle during the first part of the journey?
(ii) How far does the particle travel in the first 12 seconds of its motion?
(iii) Estimate the distance travelled in the final 5 seconds of the motion.
(iv) What is the total distance travelled by the particle?
(v) What is the final displacement of the particle?
(vi) Find the average speed of the particle.
(vii) Find the average velocity of the particle.

**Solution**

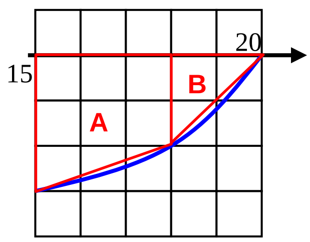
(i) During the first part of the journey the velocity of the particle increases from 0 to 4 ms$^{-1}$ in 5 seconds.

$$\text{Acceleration } = \frac{\text{change in velocity}}{\text{time}}$$
$$= \frac{4-0}{5}$$
$$= 0.8$$

Acceleration = 0.8 ms$^{-2}$.

(ii) Distance travelled = area under graph between $t$ = 0 and $t$ = 12.
Area under graph between $t$ = 0 and $t$ = 5 is $\frac{1}{2}\times 5\times 4 = 10$
Area under graph between $t$ = 5 and $t$ = 8 is $3\times 4 = 12$
Area under graph between $t$ = 8 and $t$ = 12 is $\frac{1}{2}\times 4\times 4 = 8$
Total area = 10 + 12 + 8 = 30
Distance travelled = 30 m.

(iii) Splitting up the shape as shown in the diagram:
Area of trapezium A $= \frac{1}{2}(3+2)\times 3 = 7.5$
Area of triangle B $= \frac{1}{2}\times 2\times 2 = 2$
Total area = 9.5



Distance travelled $\approx$ 9.5 m.

(iv) Distance travelled between $t$ = 12 and $t$ = 15 is $\frac{1}{2}\times 3\times 3 = 4.5$
Total distance travelled = 30 + 4.5 + 9.5 = 44 m.

(v) Final displacement = 30 – (4.5 + 9.5) = 16 m.

(vi) Average speed $= \dfrac{\text{total distance}}{\text{time}}$

$$= \frac{44}{20} = 2.2 \text{ ms}^{-1}$$

(vi) Average velocity $= \dfrac{\text{total displacement}}{\text{time}}$

$$= \frac{16}{20} = 0.8 \text{ ms}^{-1}$$