Section 3: The constant acceleration formulae

Notes and Examples

These notes contain subsections on

The constant acceleration formulae

All the equations used in this section are derived from the two properties of the velocity - time graph that you have used in the previous chapter:

- The gradient of the graph gives you the acceleration,

  $$a = \frac{v-u}{t}$$

  which can be rearranged to give the more usual form

  $$v = u + at$$

- The area under the graph, gives you the displacement

  $$s = \tfrac{1}{2}(u+v)t$$

In these equations

u = initial velocity
v = final velocity
a = acceleration
s = displacement
t = time

From the above two equations, three others can be derived.

These *constant acceleration formulae* or *suvat* equations are:

$$v = u + at$$
$$s = \tfrac{1}{2}(u+v)t$$
$$v^2 = u^2 + 2as$$
$$s = ut + \tfrac{1}{2}at^2$$
$$s = vt - \tfrac{1}{2}at^2$$

These equations are very important. **You should memorise them.**
Each equation includes 4 of the 5 variables, and each variable is missing from only one of the five equations.

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Using the constant acceleration formulae

To do the questions it is usually best to start by writing out a list of the variables and filling in the ones that you know. In simple problems you will be given details of three of the variables have to find a fourth. The easiest way to solve such problems is to choose the correct *suvat* equation. This will be the equation that involves all of the variables that you are told in the question, together with the variable that you wish to calculate.

Example 1

A particle is accelerating at a constant 6 ms^{-2}. After 8 seconds its displacement is 5 m.

(i) What is its velocity after 8 seconds?

(ii) What was its initial velocity?

(iii) Describe the motion of the particle over these 8 seconds.

Solution

Using the information in the question you can write:

s = 5 m
u = ?
v = ?
a = 6 ms^{-2}
t = 8 s

(i) You know *s*, *a* and *t* and wish to know *v*. The *suvat* equation involving *s*, *a*, *t* and *v* is: $s = vt - \frac{1}{2}at^2$, so substituting in the appropriate values gives

$$\begin{aligned} 5 = 8v - \frac{1}{2}\times 6\times 8^2 &\Rightarrow 5 = 8v - 192 \\ &\Rightarrow 197 = 8v \\ &\Rightarrow \frac{197}{8} = v \\ &\Rightarrow v = 24.6\text{ms}^{-1} \ \text{(3s.f.)} \end{aligned}$$

Note: As a general rule, round your answers to 3 significant figures and state clearly that you have done so. Avoid rounding until your final answer, to avoid rounding errors.

(ii) You know s, a and t and wish to know u. The *suvat* equation involving s, a, t and u is: $s = ut + \frac{1}{2}at^2$, so substituting in the appropriate values gives

$$\begin{aligned} 5 = 8u + \tfrac{1}{2} \times 6 \times 8^2 &\Rightarrow 5 = 8u + 192 \\ &\Rightarrow -187 = 8u \\ &\Rightarrow \frac{-187}{8} = u \\ &\Rightarrow u = -23.4 \text{ ms}^{-1} \text{ (3s.f.)} \end{aligned}$$

(iii) Initially the particle has a speed of 23.4 ms^{-1} (3s.f.) and is moving in the negative direction. Acceleration is constant at 6 ms^{-1}. During the next 8 seconds the particle slows down until it is momentarily stationary and then speeds up to reach a speed of 24.6 ms^{-1} in the positive direction. At this time its displacement is 5 m in the positive direction from its starting point.

IMPORTANT

Important note: Remember that the *suvat* equations can only be applied in situations where the acceleration is constant.

Motion under gravity

When an object is dropped or thrown vertically upwards or downwards, it can be modelled as having constant acceleration. The acceleration due to gravity is usually taken as 9.8 ms^{-2}.

To model vertical motion under gravity as having constant acceleration, you are making some assumptions. For example, you are ignoring the effect of air resistance. For a very small object, that is usually a reasonable assumption. Often, the word ‘particle’ is used, which means that the size of the object is negligible, with all its mass concentrated at a single point.

It is essential in these problems to decide at the start of each problem which direction you will take as positive. In the case of an object being dropped or thrown downwards, it makes sense to take downwards as positive since all the motion is downwards. The acceleration, velocity and displacement are then all positive throughout the motion.

In cases where an object is thrown vertically upwards, either direction could be taken as positive, although it is more usual to take upwards as positive in such cases. Once you have decided on the positive direction, then you should write down any values of u, v, a, s and t that you know, being very careful to use the correct sign. If you take upwards as positive then u will be positive but a will be negative. The signs of v and s may be positive or negative depending on where the end of the flight (or the part of the flight you are considering) is. For example, if the final position of an object is below the point of projection, the final displacement is negative.

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In questions in which a particle is projected vertically upwards, you may be interested in the maximum height reached by the particle.

The important thing to remember is that when the particle is at its greatest height, its speed is instantaneously zero. It can also be useful to remember that if the particle finishes its flight at the same point from which it was projected, then the time taken to reach the greatest height is half the total time of the flight.

Example 2
A ball is projected vertically upwards and hits the ground at the point of projection 8 seconds later. Find the speed of projection and the greatest height reached by the ball.

Solution
When the ball hits the ground, its displacement is zero.
Taking upwards to be positive:

$s = 0$	$s = ut + \frac{1}{2}at^2$
$a = -9.8$	$0 = 8u + \frac{1}{2} \times -9.8 \times 64$
$t = 8$	$u = 39.2$
$u = ?$	

The speed of projection is 39.2 ms^{-1}.
At the greatest height, the speed of the ball is zero.

$s = ?$	$v^2 = u^2 + 2as$
$a = -9.8$	$0 = 39.2^2 + 2 \times -9.8s$
$u = 39.2$	$s = 78.4$
$v = 0$	

You could also use $t = 4$ and one of the other equations, which would give the same answer.

The greatest height of the ball is 78.4 m.

Examples with more than one stage

The next two examples involve a journey of two or more stages.
In questions like this, you should write down the values of u, v, a, s and t for each stage.

Remember that the final velocity of one stage is the initial velocity of the next stage and that the accelerations across the two stages are often the same (though **NOT ALWAYS**).

Example 3
A cyclist reaches the top of a hill moving at 2 ms^{-1}, and accelerates uniformly so that during the sixth second after reaching the top he travels 13 m. Find his speed at the end of the sixth second.

Solution

The cyclist's displacement has increased by 13 m during the sixth second.

In the first 5 seconds	In the first 6 seconds
$u = 2$ ms^{-1}	$u = 2$ ms^{-1}
$v = ?$	$v = ?$
$a = a$ ms^{-2}	$a = a$ ms^{-2}
$s = x$ m	$s = (x + 13)$ m
$t = 5$ s	$t = 6$

13 m more than after 5 seconds

Using the equation which relates u, a, s and t, $s = ut + \frac{1}{2}at^2$

For $t = 5$: $x = 2 \times 5 + 12.5a$

For $t = 6$: $x + 13 = 2 \times 6 + 18a$

Solving these equations simultaneously gives $a = 2$ and $x = 35$.

To find the speed after 6 seconds:
$$\begin{aligned} v &= u + at \\ &= 2 + 2 \times 6 \\ &= 14 \end{aligned}$$
The speed at the end of the sixth second is 14 ms^{-1}.

In the example above, modelling the motion by constant acceleration may not be very realistic. However, if the road is flat and the cyclist pedals at a fairly constant rate, it may be quite a good approximation.

In Example 4, two methods of solution are given. Make sure you understand both.

Example 4

A train travels from rest at station A to stop at station B, a distance of 2100 m. For the first 20 seconds it accelerates steadily, reaching a speed of 25 ms^{-1}. It maintains this speed until the brakes are applied and the train brought to rest with uniform deceleration over the last 125 m.

Find the deceleration and the total time for the journey between the two stations.

Solution 1 (Using equations only)

Consider the three stages of the journey separately:

Stage 1 (accelerating)	Stage 2 (constant speed)	Stage 3 (decelerating)
$s = x$	$s = y$	$s = 125$
$u = 0$	$u = 25$	$u = 25$
$v = 25$	$v = 25$	$v = 0$
$a = a$	$a = 0$	$a = b$
$t = 20$	$t = t_1$	$t = t_2$

Also, from the total distance travelled we know that $x + y + 125 = 2100$.

Stage 1:
To find a use $v = u + at$: $25 = 0 + 20a$
$a = 1.25$

To find x use $s = \frac{1}{2}(u+v)t$, giving $x = \frac{1}{2}(0+25)\times 20 \Rightarrow x = 250$

Stage 2:
$250 + y + 125 = 2100 \Rightarrow y = 1725$ m

Using $s = ut + \frac{1}{2}at^2 \Rightarrow 1725 = 25t_1 + 0 \Rightarrow t_1 = \frac{1725}{25} = 69$

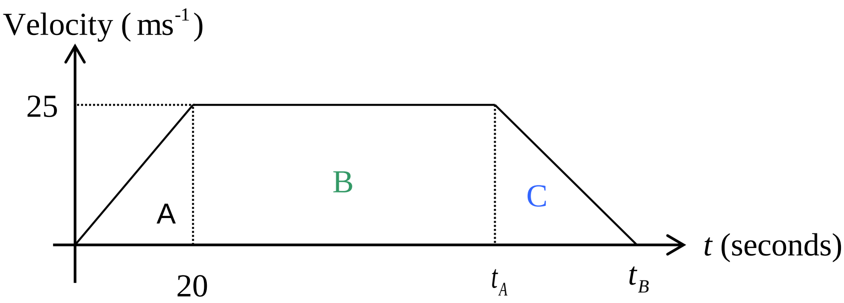
Stage 3:
To find b, use $v^2 = u^2 + 2as \Rightarrow 0 = 625 + 2\times 125b \Rightarrow b = \frac{-625}{250} = -2.5$

and to find t_2 use $s = \frac{1}{2}(u+v)t \Rightarrow 125 = \frac{1}{2}(25+0)t_2 \Rightarrow t_2 = \frac{250}{25} = 10$

(Use this equation rather than $v = u + at$ as it uses all given information rather than the quantities you have calculated – this is safer as you may have made errors in your calculations whereas given information must be assumed to be correct – this is a good general rule to follow for such questions)

The deceleration is 2.5 ms^{-2}.
The total time for the journey is 20 + 69 + 10 = 99 seconds

Solution 2 (Using a velocity time graph)
Begin by sketching a graph and marking on it what you know:



Area A + B + C = 2100 (area under graph = distance travelled)
C = 125 (from question), A = 250 (using area of triangle), so:
B = 2100 – 125 – 250 = 1725.

So $(t_A - 20)\times 25 = 1725$ (using area of rectangle B)

$\Rightarrow t_A = 89$

So $\frac{1}{2}(t_B - 89)\times 25 = 125$ (Using area of triangle C)

$\Rightarrow t_B = \dfrac{2\times 125}{25} + 89 = 99$

So total time between the stations is 99 seconds.

Acceleration = gradient of last line = $\dfrac{-25}{10} = -2.5\,\text{ms}^{-2}$, so deceleration is $2.5\,\text{ms}^{-2}$.