Section 1: Using calculus

Notes and Examples

These notes contain subsections on:

- **Using differentiation**
- **Using integration**

Using differentiation

If you are given a formula for the position of a particle in terms of t, then:

- to find its velocity at any instant, you differentiate the position with respect to time (t) and substitute in the appropriate value for t.

- to find its acceleration at any instant, you differentiate the velocity with respect to time (t) and substitute in the appropriate value for t.

Example 1

The position, s m, of a particle after t seconds is given by $s = t^3 - 5t^2 + 7t - 3$.

(a) Find (i) the velocity
(ii) the acceleration
of the particle after 3 seconds.

(b) Find t when (i) $v = 5$ ms^{-1}
(ii) $a = 6$ ms^{-2}.

Solution

(a) (i) The velocity is given by $v = \frac{ds}{dt} = 3t^2 - 10t + 7$

When $t = 3$, $v = 3\times 3^2 - 10\times 3 + 7$

$= 4$

The velocity of the particle is 4 ms^{-1}.

(ii) The acceleration is given by $a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = 6t - 10$

When $t = 3$, $a = 6\times 3 - 10$

$= 8$

The acceleration of the particle is 8 ms^{-2}.

(b) (i) From (a), $v = 3t^2 - 10t + 7$.

When $v = 5$: $3t^2 - 10t + 7 = 5$

$3t^2 - 10t + 2 = 0$

$t = 3.12$ or $t = 0.214$ (3 s.f.)

(ii) From (a), $a = 6t - 10$
When $a = 6$: $6t - 10 = 6$

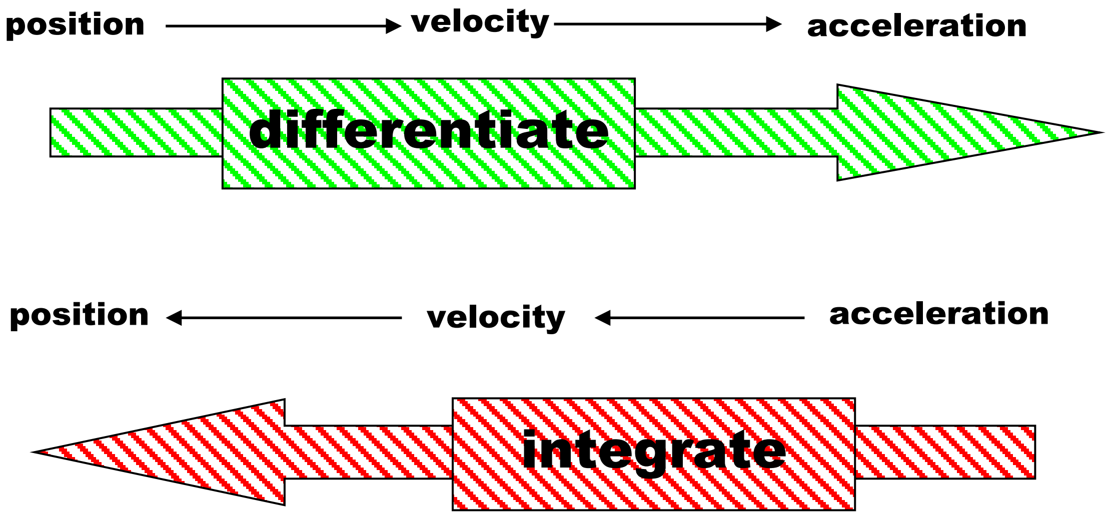
$$t = \frac{16}{6} = 2.67 \text{ (3 s.f.)}$$

Using integration

If you are given the formula for the acceleration of a particle in terms of t, then:

- To find its velocity at any instant, you integrate the acceleration with respect to time (t) and substitute in the appropriate value for t.
- To find its position at any instant, you integrate the velocity with respect to time (t) and substitute in the appropriate value for t.

This can be summarised by the diagram below:-



Example 2
A particle, initially at rest at the point where $s = 3$, has an acceleration at time t seconds given by $a = t^3 - 2t^2$.

Find expressions for its velocity and position at time t.

Solution

$$a = \frac{dv}{dt} \Rightarrow v = \int t^3 - 2t^2 dt \Rightarrow v = \frac{t^4}{4} - \frac{2t^3}{3} + c$$

To find the value of c, use the information in the question which states that the particle is initially at rest, so when $t = 0$, $v = 0$.

Substituting these into the equation for v gives $c = 0$

so $v = \frac{t^4}{4} - \frac{2t^3}{3}$

To find an expression for s, integrate again, and use the information from the question that $s = 3$ when $t = 0$ to find the constant of integration.

$s = \int v \, \mathrm{d}t = \int \frac{t^4}{4} - \frac{2t^3}{3} \mathrm{d}t = \frac{t^5}{20} - \frac{t^4}{6} + k$.

Since $s = 3$ when $t = 0$, $k = 3$

so $s = \frac{t^5}{20} - \frac{t^4}{6} + 3$