

Section 1: The Normal distribution

Notes and Examples

These notes contain subsections on

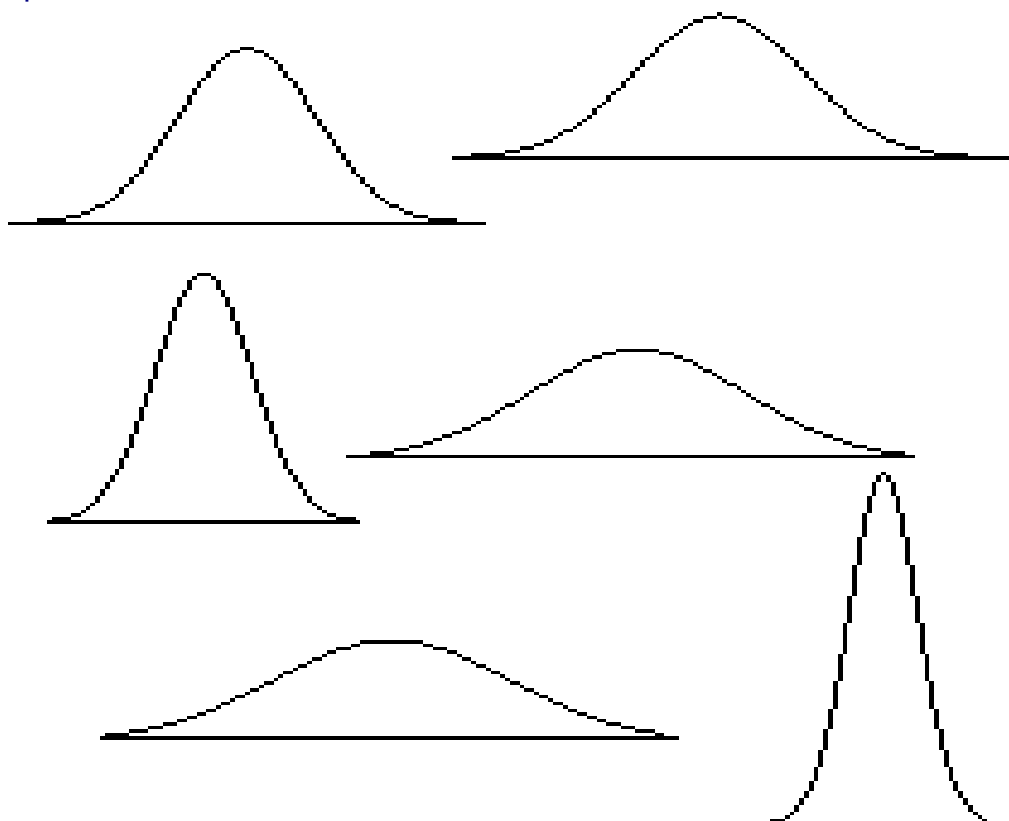
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The Normal curve

Normal distributions are a family of distributions that have the same general shape. They have a distinctive bell-shaped curve, symmetrical, with scores more concentrated in the middle than in the tails.

The Normal distribution is often illustrated by a diagram, which highlights the difference in shape because of the mean and standard deviation.

Examples of Normal curves are shown below:



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The line of symmetry of the Normal distribution is located at the mean, μ , of the distribution.

There is a non-stationary point of inflection each side of the mean. This is located one standard deviation (σ) from the mean.

The standard Normal distribution

The standard normal distribution is a normal distribution with a mean of 0 and a standard deviation of 1. Z is used if a variable has a standard normal distribution.

Any Normal distribution can be transformed to standard normal distributions.

If the variable X has mean μ and standard deviation σ , then x , a particular value of X , is transformed into z by the formula:

$$z = \frac{x - \mu}{\sigma}$$

This represents a horizontal translation through μ (so that the mean of the distribution becomes 0) and a horizontal stretch scale factor σ (so that the standard deviation of the distribution becomes 1).

Once you have standardised a normal variable the z score shows how many standard deviations above or below the mean a particular score is.

For example, consider a student who scored 80 on a test, where the test scores have a Normal distribution with a mean of 60 and a standard deviation of 10. Converting the test scores to z scores, the value x becomes:

$$z = \frac{80 - 60}{10} = 2$$

So, a z score of 2 means that the original score was 2 standard deviations above the mean.

A Normal distribution with mean μ and standard deviation σ is denoted by $N(\mu, \sigma^2)$. (Note that the second number given is the variance, σ^2 , rather than the standard deviation, σ).

Finding probabilities

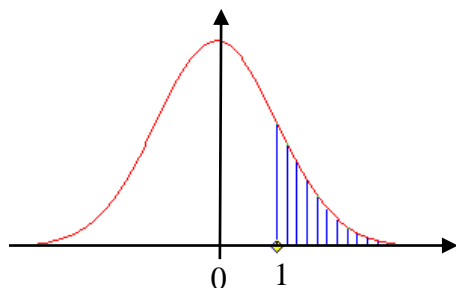
A Normal distribution is a continuous distribution so it is not possible to find the probability that a random variable takes a specific value. For example, $P(Z = 2)$ is infinitesimally small.

Instead, you usually need to find the probability that the random variable takes a value within a range, e.g. finding $P(Z < 2)$, or $P(Z > 2)$, or $P(-2 < Z < 2)$. (Note that there is no distinction between $<$ and \leq in this context).

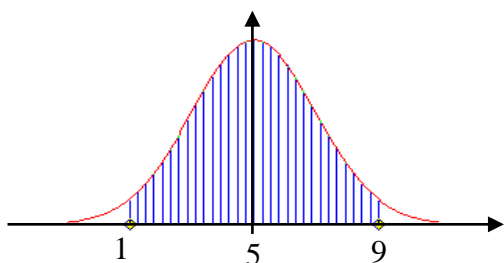
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You can find probabilities like this using your calculator statistical functions. You enter the mean and standard deviation for the distribution, and the lower and upper bound for the probabilities as required.

This diagram shows $P(Z > 1)$ for a standardised Normal distribution $N(0, 1)$. $P(Z > 1) = 0.1587$. Check that you can find this on your calculator.



This diagram shows $P(1 < X < 9)$ for a Normal distribution $N(5, 2^2)$.



$P(1 < X < 9) = 0.9544$. Check that you can find this on your calculator.

In this last example you can see that just over 95% of the values lie within 2 standard deviations from the mean for a normal variable.

Working backwards

If you need to find the value of z associated with a particular probability, then you need to work backwards. On your calculator you can do this using the Inverse normal distribution.

You will probably need to enter a value for $P(X < a)$, and then the calculator will give you the value of a . This means that if you are given $P(X > a)$ instead, you will need to do a little manipulation first.

For example, if you are given that for $X \sim N(2, 0.2^2)$, $P(X < a) = 0.15$, you can enter this into a calculator and use the inverse Normal function to obtain $a = 1.793$. So $P(X < 1.793) = 0.15$.

However, if you are given that for $Y \sim N(1, 3^2)$, $P(Y > b) = 0.6$, you first need to deduce that $P(Y < b) = 1 - P(Y > b) = 1 - 0.6 = 0.4$. You can then use your calculator to find that $b = 0.2400$. So $P(Y > 0.2400) = 0.6$.

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Using a continuity correction

When dealing with some real-life situations, you may be working with variables which are discrete, but can be modelled as continuous and has a similar shape to a Normal distribution. In cases like these you can use the Normal distribution as a model, but you often need to use a continuity correction.

For example, when working with test scores, if you want to find the probability that a student scores 80% or more, you need to calculate the probability that the score is more than 79.5%. However if you want the probability that the student scores **more than** 80% (i.e. 81% or more), you will need to calculate the probability that the score is more than 80.5%. So the wording is very important!

This is illustrated in the next example.



Example 1

A test is normally distributed with a mean of 65 and a standard deviation of 10.

- (i) What proportion of the scores are above 85?
- (ii) What proportion of the scores are between 70 and 85 inclusive?

Solution

Let X be the distribution of test scores.

$$X \sim N(65, 10^2)$$

Using a calculator function:

- (i) $P(X > 85.5) = 0.0202$
- (ii) $P(69.5 < X < 85.5) = 0.3062$



The next example involves working backwards with a continuity correction.



Example 2

Assume a test is normally distributed with a mean of 50 and a standard deviation of 6.

Let X be the distribution of test scores.

- (i) Grade A is to be awarded, as near as possible, to the top 15% of the candidates. Find the minimum score needed for a grade A.
- (ii) 14% of the candidates fail the test. What was the pass mark?

Solution

$$X \sim N(50, 6^2)$$

- (i) Let a be the minimum score needed for a grade A,
so $P(X > a - 0.5) = 0.15 \Rightarrow P(X < a - 0.5) = 0.85$
Using a calculator gives $a - 0.5 = 56.2$ so $a = 56.7$.
The score for an A should be set as 57.



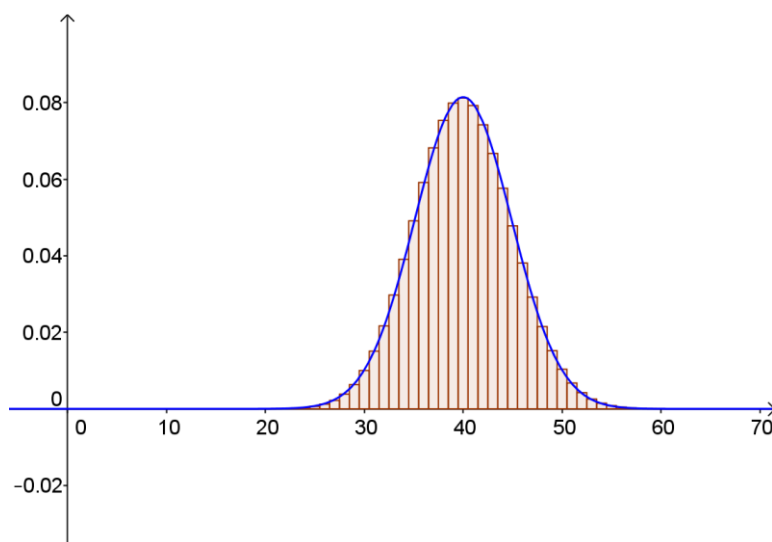
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- (ii) Let b be the pass mark.
 $P(Z < b - 0.5) = 0.14$
Using a calculator gives $b - 0.5 = 43.5$ so $b = 44$
The pass mark is 44.

Modelling the binomial distribution with a Normal distribution

If a binomial distribution $B(n, p)$ is reasonably symmetrical (i.e. p is close to 0.5), and n is large, the Normal distribution is a good approximation to the binomial distribution.

In the graph below, the binomial distribution $B(100, 0.4)$ is shown. The blue line is the Normal distribution $N(40, 24)$. You can see that the match is quite close.



Finding unknown mean and standard deviation

The next example shows how you can find an unknown mean or standard deviation, using given probability information.



Example 3

Assume a test is normally distributed with a mean of 50 and a standard deviation of σ . Let X be the distribution of test scores.
Find σ if the probability of getting a score above 66 is 0.112.

Solution 1

$$X \sim N(50, \sigma^2)$$

Using a continuity correction, $P(X > 66.5) = 0.112 \Rightarrow P(X < 66.5) = 0.888$

Using a calculator, for the standard distribution $N(0, 1)$, $P(Z < 1.216) = 0.888$

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So $z = 1.216$

Now when $x = 66.5$, $z = \frac{66.5 - 50}{\sigma} = \frac{16.5}{\sigma}$

$$1.216 = \frac{16.5}{\sigma}$$

$$\sigma = \frac{16.5}{1.216} = 13.57$$