Section 2: Probability distributions

Notes and Examples

These notes contain subsections on:

Definitions and notation

If a variable has an associated probability, (for example, the outcome when throwing a die), then the variable is referred to as a **random variable**.

A **discrete random variable** is a variable for which a list of possible numerical values can be made. A discrete random variable is usually denoted by an upper case letter, such as X, Y, or Z etc. You may think of this as the name of the variable. The particular values the variable takes are denoted by lower case letters, such as x, y, z or x_1, x_2, x_3 etc.

So for example $\mathrm{P}(X = x_1) = \frac{1}{3}$ should be read as: "The probability that the random variable X takes the value x_1 is $\frac{1}{3}$".

Probability distributions

If the discrete random variable X can take the possible values x_1, x_2 …… x_n. with probabilities p_1, p_2, ……p_n respectively then $p_1 + p_2 + \ldots\ldots + p_n = 1$. This is called a **probability distribution**.

It is useful to tabulate the possible outcomes and associated probabilities. The example below is a trivial one which serves to illustrate the correct notation.

Example 1

A fair die is thrown. The number shown on the die is the random variable X.
Tabulate the possible outcomes.

Solution

X takes the six possible outcomes 1, 2, 3, 4, 5, 6 which each have probability $\frac{1}{6}$.

r	1	2	3	4	5	6
$\mathrm{P}(X = r)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

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A probability distribution can be illustrated using a vertical line chart.

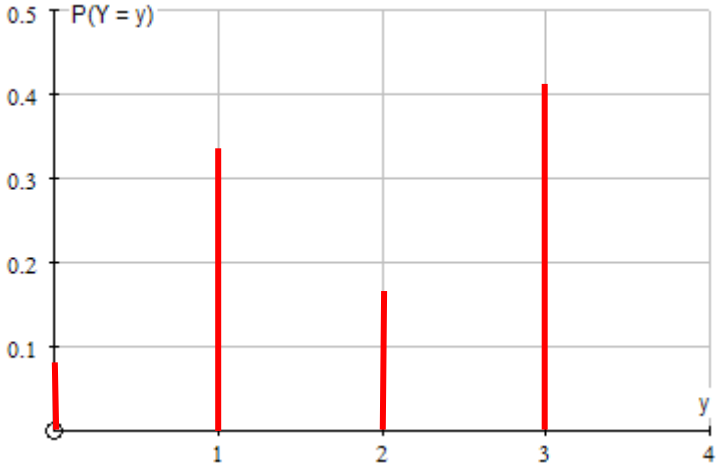
Example 2

Y takes the possible outcomes 0, 1, 2, 3 with probabilities $\frac{1}{12}$, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{5}{12}$ respectively.
Draw a diagram to illustrate the probability distribution of Y.

Solution

y	0	1	2	3
$P(Y = y)$	$\frac{1}{12}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{5}{12}$

Note that $\frac{1}{12}+\frac{1}{3}+\frac{1}{6}+\frac{5}{12}=1$



Sometimes some work is needed to find the values of the probabilities.

Example 3

Two unbiased spinners, one numbered 1, 3, 5, 7 and the other numbered 1, 2, 3 are spun. The random variable X is the sum of the two results.
Find the probability distribution for X.

Solution

Listing all the possible outcomes is best done in a table.

		1st spinner			
		1	3	5	7
2nd spinner	1	2	4	6	8
	2	3	5	7	9
	3	4	6	8	10

Check that the probabilities add up to 1

The probability distribution for X can now be tabulated.

x	2	3	4	5	6	7	8	9	10
$P(X = x)$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{2}{12}$	$\frac{1}{12}$	$\frac{2}{12}$	$\frac{1}{12}$	$\frac{2}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

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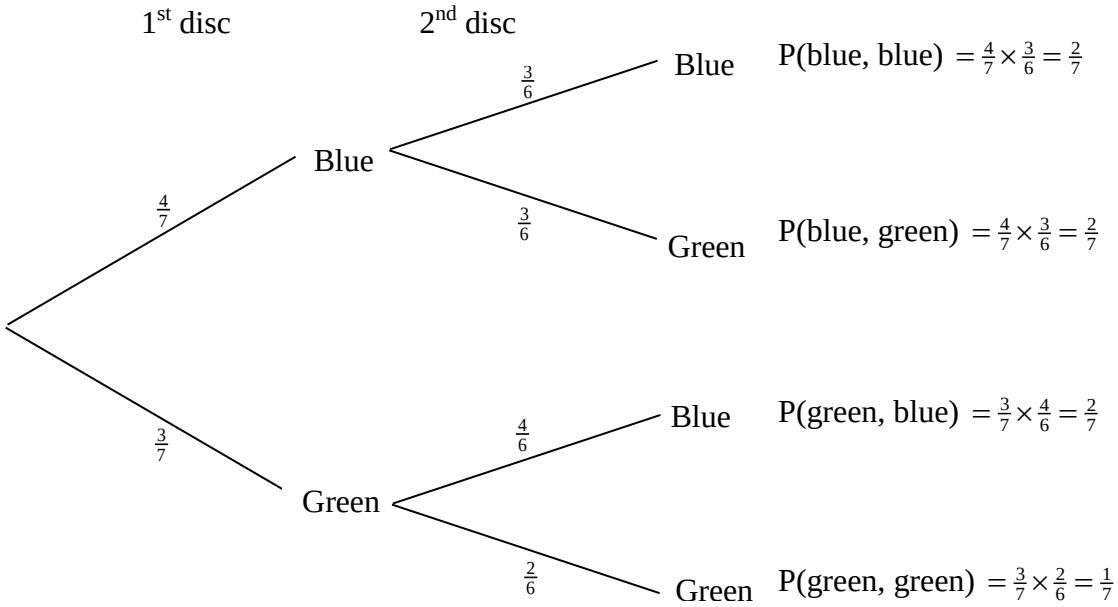
In the next example you need to use a tree diagram.

Example 4
A bag contains 4 blue discs and 3 green discs. Two discs are removed without replacement. The random variable X is the number of blue discs removed.
Find the probability distribution of X.

Solution



When $X = 0$, both discs are green, so $P(X = 0) = \frac{1}{7}$.
When $X = 1$, one of the discs is blue, so $P(X = 1) = \frac{2}{7} + \frac{2}{7} = \frac{4}{7}$.
When $X = 2$, both discs are blue, so $P(X = 2) = \frac{2}{7}$.

x	0	1	2
$P(X = x)$	$\frac{1}{7}$	$\frac{4}{7}$	$\frac{2}{7}$

Probability distributions defined algebraically

It is often convenient to define the probability distribution by writing it as an algebraic function.

Example 5
The probability distribution of a random variable X is given by:

$$P(X = r) = \frac{r}{15} \text{ for } r = 1, 3, 4, 7.$$

Tabulate the possible outcomes.

Solution

$r = 1$ $\quad$ $P(X = 1) = \frac{1}{15}$

$r = 3$ $\quad$ $P(X = 3) = \frac{3}{15} = \frac{1}{5}$

$r = 4$ $\quad$ $P(X = 4) = \frac{4}{15}$

$r = 7$ $\quad$ $P(X = 7) = \frac{7}{15}$

r	1	3	4	7
$P(X = r)$	$\frac{1}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{7}{15}$

Check: $\frac{1}{15}+\frac{3}{15}+\frac{4}{15}+\frac{7}{15}=1$

Sometimes the probability distribution will be defined in terms of a constant.

Example 6

The probability distribution of a random variable Y is given by:

$P(Y = y) = cy$ for $y = 1, 2, 3, 4$

Find the value of c and tabulate the probability distribution.

Solution

$y = 1$ $\quad$ $P(Y = 1) = c \times 1 = c$

$y = 2$ $\quad$ $P(Y = 2) = c \times 2 = 2c$ etc

y	1	2	3	4
$P(Y = y)$	c	$2c$	$3c$	$4c$

Since the probabilities must add up to 1: $\quad c+2c+3c+4c=1$

$$10c=1$$

$$c=\frac{1}{10}$$

y	1	2	3	4
$P(Y = y)$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$

The discrete uniform distribution

A special probability distribution is the discrete uniform distribution, in which there are a number of equally likely outcomes. You have of course worked with this distribution many times, when dealing with dice throws, random numbers and so on!

Example 1 in these notes shows a discrete uniform distribution.