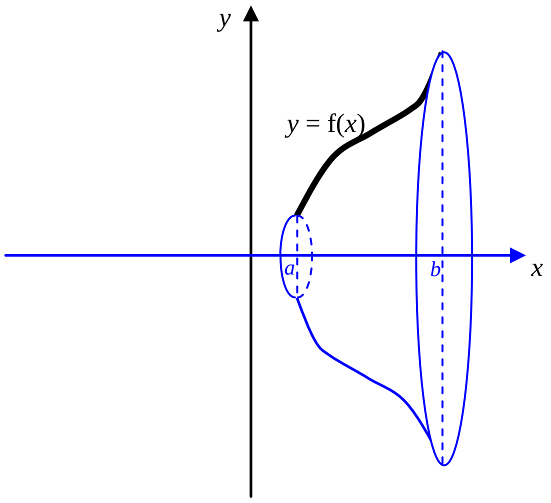
Section 1: Volumes of revolution

Notes and Examples

These notes contain subsections on

- Solids of revolution formed by rotation about the x-axis
- Solids of revolution formed by rotation about the y-axis

Solids of revolution formed by rotation about the x-axis



The diagram above shows the solid of revolution formed when the section of the curve $y = \mathrm{f}(x)$ between $x = a$ and $x = b$ is rotated through 360° about the x−axis.

The volume of the solid is given by

$$V = \int_a^b \pi y^2 \, \mathrm{d}x$$

Example 1

A solid is formed by rotating the part of the graph of $y = 2x^2$ between $x = 1$ and $x = 2$ through 360° about the x-axis.
Find the volume of the solid.

Solution

$$\text{Volume} = \int_1^2 \pi y^2 \mathrm{d}x$$

$$= \int_1^2 \pi (2x^2)^2 \mathrm{d}x$$

Substitute $y = 2x^2$ into the formula

$$= \pi \int_1^2 4x^4 \mathrm{d}x$$

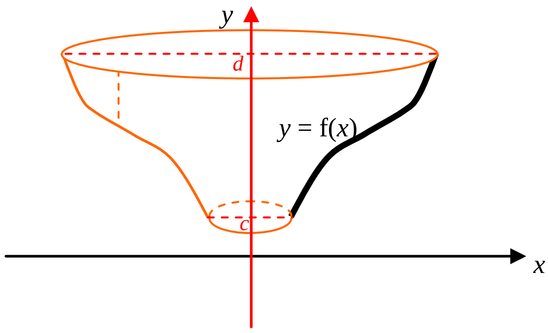
$= \pi \left[\frac{4}{5} x^5 \right]_1^2$

$= \frac{128}{5}\pi - \frac{4}{5}\pi$

$= \frac{124}{5}\pi$

It is a good idea to leave your answer as a multiple of π

Solids of revolution formed by rotation about the *y*-axis



The diagram above shows the solid of revolution formed when the section of the curve $y = f(x)$ between $y = c$ and $y = d$ is rotated through 360° about the y−axis.

The volume of the solid is given by

$$V = \int_c^d \pi x^2 \, dy$$

IMPORTANT

Notice that in this case the integration is carried out with respect to y rather than x and the limits of integration are the y-coordinates rather than the x-coordinates.

Example 2

A solid is formed by rotating the part of the graph of $y = 2x^2$ between $x = 1$ and $x = 2$ through 360° about the y-axis.

Find the volume of the solid.

Solution

$y = 2x^2 \Rightarrow x^2 = \frac{1}{2}y$

You need x in terms of y to be substituted into the formula

When $x = 1$, $y = 2$

When $x = 2$, $y = 8$

The limits of integration need to be the y-coordinates.

$\text{Volume} = \int_2^8 \pi x^2 \, dy$

$= \pi \int_2^8 \frac{1}{2} y \, dy$

Substitute $x^2 = \frac{1}{2}y$ into the formula

$= \pi \left[\frac{1}{4} y^2 \right]_2^8$

$= 16\pi - \pi$

$= 15\pi$