# Section 4: Finding distances

## Exercise level 1

1. A plane has equation $x-2y+z+22=0$. The point P is (2, 1, -4).

   (i) Use the formula $\left|\dfrac{n_1\alpha+n_2\beta+n_3\gamma+d}{\sqrt{n_1^2+n_2^2+n_3^2}}\right|$ for the distance of the point $(\alpha,\beta,\gamma)$ to the plane $n_1x+n_2y+n_3z+d=0$ to find the distance of P from the plane.

   (ii) Write down an equation for the line L that passes through P and is perpendicular to the plane.

   (iii) Find the point Q where the line L intersects the plane.

   (iv) Find the distance PQ and check it is the same as the distance found in (i).

2. A line L has equation $\mathbf{r}=\begin{pmatrix}1\\0\\3\end{pmatrix}+\lambda\begin{pmatrix}-2\\3\\1\end{pmatrix}$. The point P is (-5, 5, 4).

   The point M on the line has coordinates $(1-2\lambda,\ 3\lambda,\ 3+\lambda)$.

   (i) Write down the vector $\overrightarrow{PM}$ in terms of $\lambda$.

   (ii) Find the value of $\lambda$ for which $\overrightarrow{PM}$ is perpendicular to the line L.

   (iii) Hence find the coordinates of M.

   (iv) Hence find the shortest distance of the point P from the line L.

3. A line $L_1$ has equation $\mathbf{r}=\begin{pmatrix}10\\3\\-13\end{pmatrix}+\lambda\begin{pmatrix}5\\2\\-3\end{pmatrix}$.

   (i) Write down an equation of the line $L_2$ that is parallel to $L_1$ and passes through the point (2, 3, -1).

   (ii) Use the method of Question 2 to find the distance between the two parallel lines by finding the distance of the point (2, 3, -1) from the line $L_1$.

4. Two lines $L_1$ and $L_2$ have equations $\mathbf{r}=\begin{pmatrix}-8\\-13\\28\end{pmatrix}+\lambda\begin{pmatrix}2\\-3\\1\end{pmatrix}$ and $\mathbf{r}=\begin{pmatrix}11\\-4\\-15\end{pmatrix}+\mu\begin{pmatrix}1\\4\\2\end{pmatrix}$ respectively. The point P on $L_1$ has coordinates $(-8+2\lambda,-13-3\lambda,28+\lambda)$ and Q on $L_2$ has coordinates $(11+\mu,-4+4\mu,-15+2\mu)$. The lines are skew.

   (i) Write down the vector $\overrightarrow{PQ}$.

   (ii) $\overrightarrow{PQ}$ is perpendicular to the line $L_1$. Find an equation connecting $\lambda$ and $\mu$.

   (iii) $\overrightarrow{PQ}$ is also perpendicular to the line $L_2$. Find a second equation connecting $\lambda$ and $\mu$.

   (iv) Solve your equations from (ii) and (iii) to find the values of $\lambda$ and $\mu$.

   (v) Hence write down the coordinates of P and Q and find the distance between the two lines.