Section 3: The equation of a plane

Exercise level 2

1. (i) Find, in the form $\mathbf{r}.\mathbf{n} = d$, the equation of the plane which contains the point (2, -6, 1) and is perpendicular to the vector $3\mathbf{i} + \mathbf{j} + 3\mathbf{k}$.
   (ii) Write the equation in Cartesian form.
   (iii) Find where the plane and the line $\mathbf{r} = \mathbf{j} - 2\mathbf{k} + \lambda(2\mathbf{i} - 5\mathbf{j} + \mathbf{k})$ intersect.
   (iv) Find the angle between the line and the plane.

2. (i) Find the position vector of the point of intersection of the line
   $\mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}$ and the plane $\mathbf{r}.\begin{pmatrix} -5 \\ 1 \\ -7 \end{pmatrix} = 9$.
   (ii) Find the angle between the line and the plane.

3. A plane contains the points A (3, 0, 2), B (1, -1, 1) and C (2, 3, -1).
   Find the equation of the plane
   (i) in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$
   (ii) in the form $\mathbf{r}.\mathbf{n} = d$

4. Show that the plane $\mathbf{r}.(3\mathbf{i} - 2\mathbf{j} + \mathbf{k}) = 1$ contains the line
   $\mathbf{r} = 3\mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} - \mathbf{k})$.

5. A plane has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}$.
   Write the equation of this plane in Cartesian form.