Section 3: The equation of a plane

Exercise level 1

1. Find, in Cartesian form, the equation of the plane containing the point (2, -3, 1) and perpendicular to the vector $\begin{pmatrix} 5 \\ 1 \\ -2 \end{pmatrix}$.

2. Find, in the form $\mathbf{r}.\mathbf{n} = d$, the equation of the plane containing the point (1, 4, 0) and perpendicular to the vector $\begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix}$.

3. A plane contains three points A (2, 1, 0), B (3, -1, 1) and C (1, 2, 5).
   (i) Show that the vector $\begin{pmatrix} 11 \\ 6 \\ 1 \end{pmatrix}$ is perpendicular to both $\overrightarrow{AB}$ and $\overrightarrow{BC}$.
   (ii) Hence find the equation of the plane in Cartesian form.

4. A plane contains three points P (1, 4, 2), Q (0, 1, -1) and R (3, -2, 1).
   Find an equation for the plane in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$.

5. For each of the following lines and planes, find
   (a) the point of intersection of the line and the plane
   (b) the angle between the line and the plane
   (i) $\mathbf{r} = \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$ and $2x - 3y + z = 3$
   (ii) $\mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix}$ and $3x + y + 4z = 3$
   (iii) $\mathbf{r} = \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -3 \\ 2 \end{pmatrix}$ and $5x + 2y + 7z = 11$