Section 2: The vector equation of a line

Crucial points

1. **Make sure you understand the relationship between vector and cartesian equations of lines**

   The line $\mathbf{r} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ has a gradient of $\frac{1}{3}$.

   The line $\mathrm{r} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ has a gradient of -2.

2. **Make sure you know how to find the angle between two lines**
   To find the angle between two lines simply find the angle between the two direction vectors.

3. **Remember to watch your signs when converting between the vector and cartesian equations of a line.**

   Example: The cartesian equation of the line $\mathbf{r} = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -5 \\ 1 \end{pmatrix}$

   is $\dfrac{x-(-2)}{3} = \dfrac{y-1}{-5} = \dfrac{z-3}{1}$

   which tidies up to give: $\dfrac{x+2}{3} = \dfrac{1-y}{5} = z-3$

4. **Be careful when writing down the Cartesian equation of a line which has one or two zeros in the direction vector.**

   For example, you might think that the line $\mathbf{r} = \begin{pmatrix} 2 \\ 6 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 4 \\ -3 \end{pmatrix}$ in Cartesian

   form is $\dfrac{x-2}{0} = \dfrac{y-6}{4} = \dfrac{z+2}{-3}$. However, division by 0 is undefined. For this line, the x-coordinate of all points is 2, so an alternative way to write the equation of the line is $x = 2,\ \dfrac{y-6}{4} = \dfrac{z+2}{-3}$.

5. **Make sure that you can identify points on a line correctly**
   Students often think that if a particular point lies on a line, then scalar multiples of that point also lie on the same line. This is not normally the case (except for lines passing through the origin – can you see why?)

   e.g. The point with position vector $\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$ is on the line $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ but the

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point with position vector $\begin{pmatrix}4\\6\\8\end{pmatrix}$ is not, since the equations $\begin{matrix}4=2+\lambda\\6=3+\lambda\\8=4+\lambda\end{matrix}$ cannot be solved simultaneously, so $\begin{pmatrix}4\\6\\8\end{pmatrix}$ does not satisfy the equation of the line .

The reason students often make this mistake is because it **is true** that if a line is parallel to a given vector, it will also be parallel to all scalar multiples of that vector.

6. **Make sure you use different symbols to represent the parameters in the equations of different lines**

   This must be done in order to avoid confusion. If the same symbol were used it would imply that the parameters in each line always have equal values, which is certainly not true.

   e.g. Lines l_1 and l_2 have vector equations: $\mathbf{r}=\begin{pmatrix}4\\3\\0\end{pmatrix}+\lambda\begin{pmatrix}5\\2\\3\end{pmatrix}$ and

   $\mathbf{r}=\begin{pmatrix}-5\\4\\1\end{pmatrix}+\mu\begin{pmatrix}2\\-1\\0\end{pmatrix}$.

   Different symbols, λ and μ, are used for the parameters to indicate that they are separate values.\