Section 2: The vector equation of a line

Exercise level 2

1. A, B, C, and D are the points (2, 3), (4, -1), (6, 0) and (3, 5) respectively.
   (i) Write down the vector equations of AB and CD.
   (ii) Find the position vector of the point of intersection of AB and CD.
   (iii) Find the angle between the lines AB and CD.

2. A(1, 0) B(0, 3) C(2, 5) D(3, 2) are vertices of the quadrilateral ABCD.
   (i) Write down the vector equations of AB, BC, CD, and AD.
   (ii) What sort of quadrilateral is ABCD?
   (iii) Find the internal angles of the quadrilateral.

3. Show that the lines
   $\mathbf{r_1} = -6\mathbf{j} + \lambda(\mathbf{i} + \mathbf{j})$, $\mathbf{r_2} = 9\mathbf{j} + \mu(3\mathbf{i} - 2\mathbf{j})$ and $\mathbf{r_3} = -15\mathbf{j} + \delta(\mathbf{i} + 2\mathbf{j})$
   all pass through the same point and find the coordinates of this point.

4. Find the angle between the lines
   $$\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 5 \\ 1 \end{pmatrix} \text{ and } \mathbf{s} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ 2 \\ 2 \end{pmatrix}$$

5. Show that the lines $\mathbf{r} = \mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} - 3\mathbf{k})$ and $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + \mu(3\mathbf{j} + 5\mathbf{k})$ intersect and find the coordinates of the point of intersection.

6. Decide which of the points A, B or C with position vectors $\mathbf{a} = \mathbf{i} - 2\mathbf{j}$, $\mathbf{b} = 3\mathbf{i} - \mathbf{j} - \mathbf{k}$ and $\mathbf{c} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$ lie on the line $\mathbf{r} = 2\mathbf{i} - 3\mathbf{j} + 2\mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})$.

7. The position vectors of A and B respectively are $4\mathbf{i} - 8\mathbf{j}$ and $7\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$.
   (i) Find the vector equation of the line AB.
   (ii) Given that the equation of the line CD is $\mathbf{r} = 8\mathbf{i} - 2\mathbf{j} + \mu(\mathbf{i} + \mathbf{j} - \mathbf{k})$, find the point of intersection (if any) of AB and CD.

8. The lines with equations given by
   $$\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix} \text{ and } \mathbf{s} = \begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 3 \\ k \end{pmatrix}$$
   intersect at the point P. Find the value of k and the position vector of P.