Section 2: The vector equation of a line

Exercise level 1

1. Find vector equations for the lines joining
   (i) (2, 5) to (3, -1)
   (ii) (-3, 2) to (1, 6)
   (iii)passing through (0, 6) and parallel to $3\mathbf{i} - \mathbf{j}$

2. Find the points of intersection of the lines
   (i) $\mathbf{r} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $\mathbf{s} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 2 \end{pmatrix}$
   (ii) $\mathbf{r} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 0 \end{pmatrix}$ and $\mathbf{s} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

3. (i) Find the angle between the lines
   $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + \lambda(3\mathbf{i} - 4\mathbf{j})$ and $\mathbf{s} = 3\mathbf{i} + \mathbf{j} + \mu(2\mathbf{i} - 3\mathbf{j})$.
   (ii) Which line is perpendicular to
   (a) $8\mathbf{i} + 6\mathbf{j}$ (b) $6\mathbf{i} + 4\mathbf{j}$
   (iii) For each line, find the unit vector which is parallel to the line.

4. Find the vector and Cartesian equations of the line joining (3, 1, 1) to (-2, 3, 5).

5. Write in Cartesian form the equation of the line $\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$.

6. Write in vector form the equation of the line $\frac{x-1}{1} = \frac{y+2}{4} = \frac{z-3}{2}$.

7. Find whether each pair of lines intersects or not. If they do intersect, give the coordinates of the point of intersection.
   (i) $\underset{\sim}{\mathbf{r}} = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 0 \\ -3 \end{pmatrix}$ and $\underset{\sim}{\mathbf{r}} = \begin{pmatrix} 5 \\ 3 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$
   (ii) $\underset{\sim}{\mathbf{r}} = \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$ and $\underset{\sim}{\mathbf{r}} = \begin{pmatrix} 5 \\ 1 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$
   (iii) $\underset{\sim}{\mathbf{r}} = \begin{pmatrix} 2 \\ 6 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 2 \\ -3 \end{pmatrix}$ and $\underset{\sim}{\mathbf{r}} = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ -3 \end{pmatrix}$

(iv) $\underset{\sim}{\mathbf{r}} = \begin{pmatrix} 1 \\ 2 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$ and $\underset{\sim}{\mathbf{r}} = \begin{pmatrix} 1 \\ -5 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}$